UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	CALCULUS OF VARIATION
COURSE CODE	:	BWA 31203
PROGRAMME	:	3 BWA
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

Q1 Given the functional $J\left[y(x)\right]=\int_{0}^{1}\left(x^2y'^2(x)+y^2(x)\right)dx$.

(a) Find its first variation, $\delta J\left[y(x)\right]$.

(6 marks)

(b) Find its second variation, $\delta^2 J\left[y(x)\right]$.

(6 marks)

(c) Show that the functional is continuous on the function $y_0(x)=x$.

(8 marks)

Q2 (a) Find the extremals for the following functional

$$J\left[y(x)\right]=\int_{-1}^{0}\left(240y-y'''^2\right)dx,$$

subject to the conditions

$$y(-1)=1,\ y(0)=0,\ y'(-1)=-4.5,$$
$$y'(0)=0,\ y''(-1)=16,\ y''(0)=0.$$

(16 marks)

(b) Show that there is no solution to the problem of finding a possible extremal to the functional

$$J\left[y(x)\right]=\int_{0}^{1}\sqrt{y(x)-x}\ dx,$$

with boundary conditions $y(0)=0,\ y(1)=1$ and $y(x)\geq x$ on [0, 1].

(4 marks)

Q3 Consider the functional

$$J\left[y(x)\right]=\int_{0}^{b}\left(y'^2(x)+2yy'(x)-16y^2(x)\right)dx,\ y(0)=y(b)=0.$$

(a) Find its possible extremal.

(8 marks)

(b) Find an interval of values for b so the extremal can be included in a central field of extremals. Identify its centre.

(5 marks)

(c) Determine whether the extremal provides the functional a strong minimum on $[0, b]$.

(4 marks)

(d) Investigate if J will achieve a minimum on $[0,\ b]$ if $b=\frac{\pi}{4}$. Explain your result.

(3 marks)

Q4 Find the function that will extremize $J[y(x)]=\int_0^1 \left(y'^2(x)+y(x)y'(x)\right)dx$ with boundary conditions $y(0)=1,\ y(1)=1$ when subject to the isoperimetric condition of

$$K[y(x)]=\int_0^1 \left(y(x)-y'^2(x)\right)dx=1.$$

(20 marks)

Q5 By using the direct method of Ritz, find an approximate solution to the nonlinear equation $y''+x=0$ with boundary conditions $y(0)=1$ and $y(1)=0$ and compare it with the exact solution.

(20 marks)

- END OF QUESTION -