UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS 1
COURSE CODE	:	BWM 10103 / BSM 1913
PROGRAMME	:	4 BEE/ 4 BFC/ 4 BDD
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) The function *f(x)* is defined by

$$f(x)=\begin{cases}\sin x, & 0\le x<\frac{1}{2}\pi,\\ px+3, & \frac{1}{2}\pi\le x<2\pi,\\ q, & x\ge 2\pi.\end{cases}$$

where p and q are constants. Find the value of p and q if *f(x)* is continuous at $x=\frac{1}{2}\pi$ and $x=2\pi$.

(7 marks)

(b) By using $\lim_{x\to 0}\frac{\sin x}{x}=1$, find;

(i) $\lim_{x\to 0}\frac{\sin 3x}{\tan 3x}$,

(5 marks)

(ii) $\lim_{x\to 0}\frac{6x-\sin 2x}{2x+3\sin 4x}$.

(5 marks)

(c) Evaluate

$$\lim_{x\to 2}\frac{x-4}{3-\sqrt{2x+5}}$$

(i) without using L'Hopital's rule,

(4 marks)

(ii) by using L'Hopital's rule.

(4 marks)

Q2 (a) Determine $\frac{dy}{dx}$ if given $x = \frac{3t}{1+t^3}$ and $y = \frac{3t^2}{1+t^3}$ when $t = 2$.

(6 marks)

(b) Find $\frac{dy}{dx}$ for the implicit function $x^2y + 5y^2 - 2x = 0$. Simplify your answer.

(6 marks)

(c) A curve has a parametric equations $x = at + 2at^2$ and $y = 2at^3 + 6at^4$. Show that

$$\frac{d^2y}{dx^2} = \frac{12t}{a(1+4t)}$$

(6 marks)

(d) By using the chain rule, find the derivatives of the following function.

$$y = \frac{(3x+1)^5}{(2-x)^{10}}$$

(7 marks)

Q3 (a) Find the following integrals.

(i) $\int 3x(x+2)^5 dx$,

(5 marks)

(ii) $\int \frac{\sqrt{3+\sqrt{x}}}{\sqrt{x}} dx$.

(5 marks)

(b) Evaluate $\int_2^6 x^2(x-2)^{3/2}\, dx$ by using the tabular method.

(8 marks)

(c) By using the method of integration by part, find $\int_{-\pi}^{\pi} \frac{\sin x}{(2x)^{-1}} dx$.

(7 marks)

Q4 (a) Evaluate $\frac{d}{dx}\left[\cos^{-1}\left(\frac{x-4}{8}\right)\right]$ by using implicit differentiation.

(7 marks)

(b) Determine $\int \frac{dx}{\cos x+1}$ by substituting of $t = tan\ \frac{x}{2}$ and $tan\, x = \frac{2t}{1-t^2}$.

(7 marks)

(c) By using $x = a\ sin\theta$, integrate $\int \frac{\sqrt{25-x^2}}{x^2}\, dx$.

(6 marks)

(d) Differentiate $\tan^{-1}\left(\frac{1+4x}{1-4x}\right)$ with respect to x.

(5 marks)

- END OF QUESTION -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \text{sech}^2 x \, dx = \tanh x + C$$

$$\int \text{csch}^2 x \, dx = -\coth x + C$$

$$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$$

$$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}}\, dx = \sin^{-1} x + C, \ |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}}\, dx = \cos^{-1} x + C, \ |x| < 1$$

$$\int \frac{1}{1+x^2}\, dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2}\, dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}}\, dx = \sec^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}}\, dx = \csc^{-1} x + C, \ |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}}\, dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\, dx = \cosh^{-1} x + C, \ |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}}\, dx = \text{sech}^{-1} |x| + C, \ 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}}\, dx = \text{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2}\, dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \coth^{-1} x + C, & |x| > 1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

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Formulae

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \dfrac{2t}{1+t^2}$	$\cos x = \dfrac{1-t^2}{1+t^2}$	$\sin 2x = \dfrac{2t}{1+t^2}$	$\cos 2x = \dfrac{1-t^2}{1+t^2}$
$\tan x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{2dt}{1+t^2}$	$\tan 2x = \dfrac{2t}{1-t^2}$	$dx = \dfrac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x + \sin^2 x = 1$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$2\sin ax \cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax \sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax \cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

$\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$

$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa = \frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa = \frac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{\left[\dot{x}^2 + \dot{y}^2\right]^{3/2}}$$

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$L = \int_{y_1}^{y_2} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$S = 2\pi \int_{x_1}^{x_2} f(x) \sqrt{1 + \left(\frac{d}{dx}[f(x)]\right)^2}\, dx$$

$$S = 2\pi \int_{y_1}^{y_2} g(y) \sqrt{1 + \left(\frac{d}{dy}[g(y)]\right)^2}\, dy$$