KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

PEPERIKSAAN AKHIR
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KERTAS SOALAN INI MENGANDUNGI 6 MUKA SURAT

PART A

Q1 Given the set of data:

x	1.0	1.3	1.6
$f(x)$	0.890	0.765	0.294

(a) Approximate $f(1.4)$ by using Lagrange Interpolating Polynomial.

(6 marks)

(b) Approximate $f(1.4)$ by using the appropriate Newton Difference method (either Newton Forward Difference method or Newton Backward Difference method).

(8 marks)

(c) If $f(1.5) = 0.452$ is added in the data above, approximate $f(1.4)$ using Newton Interpolating Divided-difference Method.

(6 marks)

Q2 (a) Given the first-order ordinary differential equation as below:

$$xy' - 4y = x^5 e^x \qquad \text{where } 1 \le x \le 2,$$

with initial condition $y(1) = 0$.

(i) By taking a step size of $h = 0.2$, solve the differential equation by Optimal method.

(8 marks)

(ii) The exact solution for the above problem is $y = x^4\left(e^x - e\right)$. Find the absolute error between the exact solution and the solution obtained in part (a) (i).

(3 marks)

(b) Given the first-order ordinary differential equation as below:

$$y' = y \cos x, \qquad \text{where } 0 \le x \le 1,$$

with initial condition $y(0) = 1$.

By taking a step size of $h = 0.2$, find the values of $y(x)$ by second order Taylor series method.

(9 marks)

PART B

Q3 (a) Solve the differential equation below

$$\frac{dy}{dx}+\left(\frac{-x}{1-x^2}\right)y=\frac{1}{1-x^2}.$$

[Hint : $\int \frac{1}{\sqrt{1-x^2}}\,dx=\sin^{-1}x+C$]

(10 marks)

(b) Solve the differential equation below by using the method of homogeneous equation.

$$xdy-ydx-\sqrt{x^2-y^2}\,dx=0.$$

[Hint : $\int \frac{1}{\sqrt{1-x^2}}\,dx=\sin^{-1}x+C$]

(10 marks)

Q4 Solve the initial value problem below by using variation of parameter.

$$y''-5y'+6y=-e^x;\quad y(0)=y'(0)=0.$$

(20 marks)

Q5 Given $\int_1^6 2+\cos(\sqrt{2x})dx$.

(a) By using the methods below, if appropriate, find the approximate value for the integral with $n=10$ subintervals.

(i) trapezoidal rule,

(ii) 1/3 Simpson's rule,

(iii) 3/8 Simpson's rule.

(15 marks)

(b) Find the exact solution for $\int_1^6 2+\cos(\sqrt{2x})dx$ by using your calculator.

Hence calculate the absolute error and relative error (in percentage) for answer in (a) (i).

(4 marks)

(c) Suggest a way to reduce the error in calculating the definite integral by trapezoidal rule.

(1mark)

Q6 Given the system of linear equations.

$$\begin{aligned} 6x_1 &+ 3x_2 & &= 6 \\ 3x_1 &+ 10x_2 &- 2x_3 &= 1 \\ & -2x_2 &+ 8x_3 &= 5 \end{aligned}$$

(a) Write the system in matrix form, $A\,x = B$. Then, show that the matrix A is positive definite by checking the following conditions:

(i) A is nonsingular (or $|A| \neq 0$),

(ii) $a_{ii} > 0, \quad \forall i = 1, 2, 3,$

(iii) $\max\limits_{\substack{1 \le k \le n \\ 1 \le j \le n}} | a_{kj} | \le \max\limits_{1 \le i \le n} | a_{ii} |,$

(iv) $(a_{ij})^2 < a_{ii} a_{jj}, \quad \forall\, i, j = 1, 2, 3.$ $_{(i \neq j)}$

(7 marks)

(b) Solve the system by Cholesky factorization method.

(13 marks)

System of Linear Equations

Crout Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & \vdots \\ l_{31} & l_{32} & l_{33} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & 1 & u_{23} & \cdots & u_{2n} \\ 0 & 0 & 1 & \cdots & u_{3n} \\ \vdots & \ddots & \ddots & \ddots & u_{4n} \\ 0 & \cdots & 0 & 0 & 1 \end{pmatrix}$$

Doolittle Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ l_{21} & 1 & 0 & \cdots & \vdots \\ l_{31} & l_{32} & 1 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \cdots & u_{3n} \\ \vdots & \ddots & \ddots & \ddots & u_{4n} \\ 0 & \cdots & 0 & 0 & u_{nn} \end{pmatrix}$$

Cholesky Factorization: $A = LU$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & \vdots \\ l_{31} & l_{32} & l_{33} & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ l_{n1} & l_{n2} & l_{n3} & \cdots & l_{nn} \end{pmatrix} \begin{pmatrix} l_{11} & l_{12} & l_{13} & \cdots & l_{1n} \\ 0 & l_{22} & l_{23} & \cdots & l_{2n} \\ 0 & 0 & l_{33} & \cdots & l_{3n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & l_{nn} \end{pmatrix}$$

Interpolation

Lagrange interpolation : $P_n(x) = \sum_{i=0}^{n} L_i(x) f_i$ for $k = 0, 1, 2, 3, \ldots, n$ with $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

Newton's interpolatory divided-difference formula:

$$P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \cdots + f_0^{[n]}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Newton forward difference Method :

$$P_n(x) = f_0 + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 f_0 + \cdots + \frac{r(r-1)(r-2)\cdots(r-n)}{n!}\Delta^n f_0$$

which $r = \dfrac{x - x_0}{h}$

Newton backward difference Method :

$$P_n(x) = f_n + r\nabla f_n + \frac{r(r+1)}{2!}\nabla^2 f_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 f_n + \cdots + \frac{r(r+1)(r+2)\cdots(r+n)}{n!}\nabla^n f_n$$

which $r = \dfrac{x - x_n}{h}$

Numerical Integration

Trapeziod rule : $\int_a^b f(x)\, dx \approx \frac{h}{2}\left(f_0 + f_n + 2\sum_{i=1}^{n-1} f_i \right)$

Simpson $\frac{1}{3}$ Rule: $\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,\text{odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,\text{even}}}^{n-2} f_i \right]$

Simpson $\frac{3}{8}$ rule : $\int_a^b f(x)\,dx \approx \frac{3}{8}h\left[\begin{array}{c} (f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) \\ + 2(f_3 + f_6 + \cdots + f_{n-3}) \end{array} \right]$

Ordinary Differential Equation
Initial Value Problem:

Taylor Series Method: $y(x_{i+1}) = y(x_i) + hy'(x_i) + \frac{h^2}{2!}y''(x_i) + \cdots + \frac{h^n}{n!}y^{(n)}(x_i)$

Optimal Method $y_{i+1} = y_i + \frac{1}{4}k_1 + \frac{3}{4}k_2$ where $k_1 = hf(x_i,\, y_i)$

$k_2 = hf(x_i + \frac{2}{3}h,\, y_i + \frac{2}{3}k_1)$