# KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

## PEPERIKSAAN AKHIR
## SEMESTER I
## SESI 2006/2007

| | | |
|---|---|---|
| NAMA MATA PELAJARAN | : | MATEMATIK KEJURUTERAAN I |
| KOD MATA PELAJARAN | : | BSM 1913 (FKEE & FKMP) |
| KURSUS | : | 1 BEM, 1 BEP, 1 BER, 1 BET, 1 BDP |
| TARIKH PEPERIKSAAN | : | NOVEMBER 2006 |
| JANGKA MASA | : | 3 JAM |
| ARAHAN | : | 1. JAWAB **SEMUA** SOALAN DALAM **BAHAGIAN A**<br>2. PILIH **TIGA (3)** SOALAN SAHAJA DALAM **BAHAGIAN B** |

KERTAS SOALAN INI MENGANDUNGI 6 MUKA SURAT

## PART A

**Q1** (a) Given $y = \dfrac{x-2}{x^2+3}$. Calculate

(i) the curvature of the curve at $x = 0$.

(ii) the radius of curvature of the curve at the point $(1, -0.25)$.

(7 marks)

(b) Find the area of the surface when the arc of $g(y) = \sqrt{16 - y^2}$ from $y = 1$ to $y = 3$ revolved about the $y$-axis.

(6 marks)

(c) Find the arc length of the curve $x = \cos^3 2t$ and $y = \sin^3 2t$ from $t = \dfrac{\pi}{4}$ to $t = \pi$.

(7 marks)

**Q2** (a) Find the Maclaurin series for $f(x) = \sin x$ up to three nonzero terms.

(3 marks)

(b) Use the result in (a) to approximate

$$\int_0^1 \frac{\sin x}{x}\,dx .$$

Write your answer in four decimal places.

(4 marks)

(c) Show that

$$1 + \frac{1\cdot 2}{1\cdot 3} + \frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5} + \frac{1\cdot 2\cdot 3\cdot 4}{1\cdot 3\cdot 5\cdot 7} + \ldots$$

converges by using ratio test.

(6 marks)

(d) Find the interval of convergence for

$$\sum_{n=1}^{+\infty} \frac{|x-2|^n}{n} .$$

(7 marks)

**PART B**

**Q3** (a) Find a nonzero value for the constant $p$ so that the function $f(x)$ will be continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{\tan px}{x}, & x < 0, \\ \dfrac{1}{2}(7x - p^2), & x \geq 0. \end{cases}$$

(7 marks)

(b) Evaluate the given limits.

(i) $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x^2}$.

(ii) $\lim\limits_{x \to \infty} \left( \dfrac{x^4 - 3x^2 + 1}{2x + 16x^4} \right)^{\frac{1}{4}}$.

(iii) $\lim\limits_{t \to \pi} (\pi - t + 1)^{\csc 2t}$.

(13 marks)

**Q4** (a) (i) Given $y = \cos^2(x + y)$. Show that

$$\frac{dy}{dx} = \frac{\sin 2(x + y)}{1 - \sin 2(x + y)}.$$

(ii) Show that $y\dfrac{d^2y}{dx^2} - \left(\dfrac{dy}{dx}\right)^2 - y^4 = 0$ if $y = \csc x$.

(13 marks)

(b) A sheet of cardboard 6 cm by 6 cm is used to make an open box by cutting squares of equal size from the four corner and folding up the sides. What dimensions will produce a box with maximum volume?

(7 marks)

**Q5** Evaluate

(a) $\int_1^e \frac{\ln x}{x}\, dx$,

(4 marks)

(b) $\int \cos 3x \sin 2x\, dx$,

(5 marks)

(c) $\int \frac{x^2+2}{x+2}\, dx$,

(5 marks)

(d) $\int \sec x \tan^3 x\, dx$.

(6 marks)

**Q6** (a) Given that $y = x\cos^{-1}\left(\frac{x}{2}\right) + \cosh^{-1}\sqrt{x^2+1}$. Find $\frac{dy}{dx}$.

(5 marks)

(b) Given that $x^2 \sinh y = 1$.

(i) Show that $\frac{dy}{dx} = -\frac{2}{x}\tanh y$.

(ii) Use the result in (b)(i) to show that

$$\frac{d^2y}{dx^2} = \frac{2}{x^2}\tanh y(\operatorname{sech}^2 y + 1).$$

(6 marks)

(c) Evaluate

(i) $\int \frac{e^x}{4-e^{2x}}\, dx$,

(ii) $\int \frac{1}{\sqrt{x^2-x}}\, dx$.

(9 marks)

## INTEGRATION

| Indefinite Integrals | Integration of Inverse Functions |
|---|---|
| $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$ | $\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \frac{1}{x}\, dx = \ln\lvert x \rvert + C$ | $\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \cos x\, dx = \sin x + C$ | $\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \sin x\, dx = -\cos x + C$ | $\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \sec^2 x\, dx = \tan x + C$ | $\int \frac{1}{\lvert x \rvert \sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \csc^2 x\, dx = -\cot x + C$ | $\int \frac{-1}{\lvert x \rvert \sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \sec x \tan x\, dx = \sec x + C$ | $\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int \csc x \cot x\, dx = -\csc x + C$ | $\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$ |
| $\int e^x dx = e^x + C$ | $\int \frac{-1}{\lvert x \rvert \sqrt{a^2 - x^2}}\, dx = \frac{1}{a}\operatorname{sech}^{-1}\left\lvert \frac{x}{a} \right\rvert + C$ |
| $\int \cosh x\, dx = \sinh x + C$ | $\int \frac{-1}{\lvert x \rvert \sqrt{a^2 + x^2}}\, dx = \frac{1}{a}\operatorname{csch}^{-1}\left\lvert \frac{x}{a} \right\rvert + C$ |
| $\int \sinh x\, dx = \cosh x + C$ | $\int \frac{1}{a^2 - x^2}\, dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert > a \end{cases}$ |
| $\int \operatorname{sech}^2 x\, dx = \tanh x + C$ | |
| $\int \operatorname{csch}^2 x\, dx = -\coth x + C$ | |
| $\int \operatorname{sech} x \tanh x\, dx = -\operatorname{sech} x + C$ | |
| $\int \operatorname{csch} x \coth x\, dx = -\operatorname{csch} x + C$ | |

## IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

### *Trigonometric Functions*

$$\cos^2 x+\sin^2 x=1$$
$$\sin 2x=2\sin x\cos x$$
$$\cos 2x=\cos^2 x-\sin^2 x$$
$$=2\cos^2 x-1$$
$$=1-2\sin^2 x$$
$$1+\tan^2 x=\sec^2 x$$
$$1+\cot^2 x=\csc^2 x$$
$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$
$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$$
$$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$$
$$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$$
$$\sin ax\cos bx=\tfrac{1}{2}[\sin(a+b)x+\sin(a-b)x]$$
$$\sin ax\sin bx=\tfrac{1}{2}[-\cos(a+b)x+\cos(a-b)x]$$
$$\cos ax\cos bx=\tfrac{1}{2}[\cos(a+b)x+\cos(a-b)x]$$

### *Hyperbolic Functions*

$$\sinh x=\frac{e^x-e^{-x}}{2}$$
$$\cosh x=\frac{e^x+e^{-x}}{2}$$
$$\cosh^2 x-\sinh^2 x=1$$
$$\sinh 2x=2\sinh x\cosh x$$
$$\cosh 2x=\cosh^2 x+\sinh^2 x$$
$$=2\cosh^2 x-1$$
$$=1+2\sinh^2 x$$
$$1-\tanh^2 x=\text{sech}^2 x$$
$$\coth^2 x-1=\text{csch}^2 x$$
$$\tanh 2x=\frac{2\tanh x}{1+\tanh^2 x}$$
$$\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$$
$$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$$
$$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$$

## SUBSTITUTIONS

| *Expression* | *Trigonometric* | *Hyperbolic* |
|---|---|---|
| $\sqrt{x^2+k^2}$ | $x=k\tan\theta$ | $x=k\sinh\theta$ |
| $\sqrt{x^2-k^2}$ | $x=k\sec\theta$ | $x=k\cosh\theta$ |
| $\sqrt{k^2-x^2}$ | $x=k\sin\theta$ | $x=k\tanh\theta$ |

$t=\tan\frac{1}{2}x$

$$\sin x=\frac{2t}{1+t^2}\qquad \cos x=\frac{1-t^2}{1+t^2}$$
$$\tan x=\frac{2t}{1-t^2}\qquad dx=\frac{2dt}{1+t^2}$$

$t=\tan x$

$$\sin 2x=\frac{2t}{1+t^2}\qquad \cos 2x=\frac{1-t^2}{1+t^2}$$
$$\tan 2x=\frac{2t}{1-t^2}\qquad dx=\frac{dt}{1+t^2}$$

## TAYLOR AND MACLAURIN SERIES

$$f(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\cdots$$
$$f(x)=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\cdots$$

## CURVATURE, ARC LENGTH AND SURFACE AREA OF REVOLUTION

$$\kappa=\frac{\left|\dfrac{d^2y}{dx^2}\right|}{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{3/2}}$$

$$\kappa=\frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{\left[\dot{x}^2+\dot{y}^2\right]^{3/2}}$$

$$L=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$L=\int_a^b\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$L=\int_c^d\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$S=2\pi\int_a^b f(x)\sqrt{1+\left(\frac{d}{dx}[f(x)]\right)^2}\,dx$$

$$S=2\pi\int_c^d g(y)\sqrt{1+\left(\frac{d}{dy}[g(y)]\right)^2}\,dy$$