KOLEJ UNIVERSITI TEKNOLOGI TUN HUSSEIN ONN

PEPERIKSAAN AKHIR
SEMESTER I
SESI 2006/2007

NAMA MATA PELAJARAN	:	MATEMATIK KEJURUTERAAN II
KOD MATA PELAJARAN	:	BSM 1923 / 1623 / 2723
KURSUS	:	1 BFA, 1 BFB, 1 BFP, 1 BKC, 1 BKJ, 1 BTD, 1 BTJ
TARIKH PEPERIKSAAN	:	NOVEMBER 2006
JANGKA MASA	:	3 JAM
ARAHAN	:	JAWAB **SEMUA SOALAN** DALAM BAHAGIAN A DAN **TIGA (3)** SOALAN DALAM BAHAGIAN B.

KERTAS SOALAN INI MENGANDUNGI 6 MUKA SURAT

PART A

Q1 (a) Solve the partial differential equation

$$\frac{\partial^2 u}{\partial x \partial t} = x^2(t-1),$$

given that at $t = 0$, $u = \cos 2x$ and at $x = 0$, $\frac{\partial u}{\partial t} = \sin t$.

(10 marks)

(b) Consider the one-dimensional heat equation.

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial u}{\partial t}$$

subject to the boundary values $u(0,t) = 50$, $u(l,t) = 100$ and the initial condition $u(x,0) = 100$. Using the method of separation of variables, show that

$$u(x,t) = (A\cos\lambda x + B\sin\lambda x)\, e^{\frac{-\lambda^2}{a^2} t}$$

where A and B are arbitrary constants, is a general solution for the heat equation.

(10 marks)

Q2 Given the function

$$f(x) = \begin{cases} 2, & x = 0 \\ 4 - x^2, & 0 < x < 2 \end{cases}$$
$$f(x) = f(x+2)$$

in which it can be written as a Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right],$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)dx,\ a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx, \text{ and } b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx.$$

(a) Sketch the graph of $f(x)$ for $0 \le x < 8$.

(4 marks)

(b) Hence, show that the Fourier series of $f(x)$ is

$$f(x) = 4\left[\frac{2}{3} + \frac{1}{\pi}\sum_{n=1}^{\infty}\left(\frac{\sin(n\pi x)}{n} - \frac{\cos(n\pi x)}{n^2\pi}\right)\right]$$

(16 marks)

PART B

Q3 (a) The R-L circuit model is given as

$$L\frac{dI}{dt}+RI=E(t)$$

where the circuit has inductance $L = 0.5$ henry, resistance $R = 5$ ohms, electromotive force $E(t) = -t$ and I is the current flowing in the circuit. The initial current is 3 Amperes.

(i) State the initial condition for the circuit.

(ii) Find the current I flowing in the circuit for all time t.

(10 marks)

(b) The equation of a moving particle is described as

$$m\frac{dv}{dt}=mg-v$$

where m is mass, g is gravity and v is the velocity of the particle. Assume that the mass is 10 kg and by neglecting the gravity, $g = 0$ m/s.

(i) Find the velocity v of the particle for all time t when the initial acceleration is 10 m/s².

(ii) From the results in (b)(i), determine the distance x of the particle for all time t when the initial distance is 1200 m.

(10 marks)

Q4 (a) By using the variation of parameters method and the indicated solution of the related homogeneous equation, find a particular integral and then a complete solution for the following non-homogeneous equation.

$$y''-y'-2y=x;\ y_1=e^{-x},\ y_2=e^{2x}$$

(16 marks)

(b) Show that the simple harmonic motion

$$m\frac{d^2u}{dt^2}+ku=0$$

has a general solution such as

$$u(t)=A\cos\sqrt{\frac{k}{m}}\,t+B\sin\sqrt{\frac{k}{m}}\,t$$

where A and B are arbitrary constants.

(4 marks)

Q5 (a) Consider the periodic function

$$f(t)=\begin{cases} e^{t}, & 0\le t<1 \\ 2(1-t), & 1\le t<2 \end{cases}$$
$$f(t)=f(t+2).$$

(i) Sketch the graph of $f(t)$ for $0\le t<10$.

(ii) Find the Laplace transform of $f(t)$.

[Hint: $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)\;dt, s>0$]

(12 marks)

(b) Consider the piecewise function

$$g(t)=\begin{cases} 1, & t<0 \\ 2t, & 0\le t<1 \\ t^2+1, & 1\le t<2 \\ t^2+t, & t\ge 2 \end{cases}$$

(i) Write the function $g(t)$ in the form of unit step function.

(ii) Find the Laplace transform of $g(t)$.

(8 marks)

Q6 (a) By using the convolution theorem, show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+9)^2}\right\}=\frac{1}{18}\left(\frac{\sin 3t}{3}-t\cos 3t\right).$$

Then, evaluate

$$\mathcal{L}^{-1}\left\{\frac{1}{[(s+2)^2+9]^2}\right\}$$

(13 marks)

(b) Solve

$$y''+2y'+2y=\delta(t-2);\; y(0)=0, y'(0)=0.$$

(7 marks)

FORMULAS

Formula – solving 1[st] ODE ($ay' + by + c = 0$)

Homogenous Equations with Constants Coefficients

General Solution	Roots
$y_h = c_1 e^{m_1 x} + c_2 e^{m_2 x}$	m_1 and m_2 are real and distinct
$y_h = (c_1 + c_2 x)e^{mx}$	m_1 and m_2 are real and equal
$y_h = e^{ax}(c_1 \cos bx + c_2 \sin bx)$	$m = a \pm bi$ is complex

Formula – solving 2[nd] order ODE ($y'' + ay' + by + c = 0$)

Non-Homogenous Equations: Method of Undetermined Coefficients

$f(x)$	y_p
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0)$
$C e^{\alpha x}$	$x^r(A e^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(A \cos\beta x + B \sin\beta x)$
$f_1(x) \pm f_2(x) \pm \ldots \pm f_n(x)$	$y_{k1} + y_{k2} + \ldots + y_{kn}$

Non-Homogenous Equations: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f}{W} dx$, $u_2 = \int \frac{y_1 f}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

Formula – trigonometry identities, basic derivatives and integrations

Trigonometry identities

$$\sin ax \cos bx = \tfrac{1}{2}[\sin(a+b)x + \sin(a-b)x]$$
$$\sin ax \sin bx = \tfrac{1}{2}[-\cos(a+b)x + \cos(a-b)x]$$
$$\cos ax \cos bx = \tfrac{1}{2}[\cos(a+b)x + \cos(a-b)x]$$

Basic derivatives and integrations

$$(uv)' = u'v + uv' \qquad \left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2}$$

$$\int u\, dv = uv - \int v\, du \qquad \int e^{au}\, du = \frac{1}{a}e^{au} + k$$

$$\int \sin au\, du = -\frac{1}{a}\cos au + k \qquad \int \cos au\, du = \frac{1}{a}\sin au + k$$

Formula – Laplace transformation

$$\textit{Definition: } \mathcal{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)dt\,,\ s > 0$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$ $(s>0)$
t	$\frac{1}{s^2}$ $(s>0)$
t^n $(n \geq 0)$	$\frac{n!}{s^{n+1}}$ $(s>0)$
t^a $(a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}}$ $(s>0)$
e^{at}	$\frac{1}{s-a}$ $(s>a)$
$\cos kt$	$\frac{s}{s^2+k^2}$ $(s>0)$
$\sin kt$	$\frac{k}{s^2+k^2}$ $(s>0)$
$\cosh kt$	$\frac{s}{s^2-k^2}$ $(s>\|k\|)$
$\sinh kt$	$\frac{k}{s^2-k^2}$ $(s>\|k\|)$
$H(t-a)$	$\frac{e^{-as}}{s}$ $(s>0)$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$
$f(t)\delta(t-a)$	$f(a)\, e^{-as}$
$f(t-a)\, H(t-a)$	$e^{-as} F(s)$
$\int_0^t f(u)g(t-u)du$	$F(s) \cdot G(s)$
$\int_0^t f(\tau)\ d\tau$	$\frac{1}{s}\ \mathcal{L}\{f(t)\}$
$y'(t)$	$s\mathcal{L}\{y(t)\} - y(0)$
$y''(t)$	$s^2\, \mathcal{L}\{y(t)\} - s\, y(0) - y'(0)$