UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	ENGINEERING MATHEMATICS II
CODE	:	BSM 1923
COURSE	:	1 BDD / 2 BDD / 3 BDD / 4 BDD 1 BFF / 2 BFF / 3 BFF / 4 BFF
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} 3+x & -\pi \le x < 0, \\ 3 & 0 \le x < \pi, \end{cases}$$
$$f(x) = f(x+2\pi).$$

(a) Sketch the graph of $f(x)$ over $-3\pi \le x \le 3\pi$.

(3 marks)

(b) Calculate the Fourier coefficient, a_0.

(4 marks)

(c) Show that

(i) $a_n = \dfrac{1}{n^2\pi}(1-\cos n\pi)$, and

(ii) $b_n = -\dfrac{\cos n\pi}{n}$,

for $n = 1, 2, 3, \ldots$

(10 marks)

(d) Hence, determine a Fourier series expansion for $f(x)$.

(3 marks)

Q2 A rod of length π m is fully insulated along its sides and its ends are dipped into ice and held at a temperature of $0^\circ C$. At $t = 0$, it has an initial temperature $x(\pi - x)^\circ C$. This heat problem is expressed by

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial t^2},$$

where $u(x,t)$ is the temperature of the rod at a distance x m from one end at time t seconds.

(a) Complete the formulation of the heat problem by filling in the blanks below.

Heat Equation: $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial t^2}$, $(0 < x < \pi, \quad t > 0)$,

Boundary Conditions: $u(0,t) =$ _______, $u(\pi,t) =$ _______, $(t > 0)$,

Initial Condition: $u(x,0) =$ _________, $(0 < x < \pi)$.

(4 marks)

(b) (i) **Show** that the solution to the heat problem is

$$u(x,t) = D_n(\sin nx)e^{-3n^2 t}$$

where D_n and n are arbitrary constants.

(ii) Also, show that its general solution is

$$u(x,t) = \sum_{n=1}^{\infty} D_n(\sin nx)e^{-3n^2 t}.$$

(6 marks)

(c) After applying the initial condition to $u(x,t)$, D_n can be evaluated by using

$$D_n = \frac{2}{\ell}\int_0^\ell x(\pi - x)\sin\left(\frac{n\pi x}{\ell}\right)dx,$$

where ℓ is the length of the rod. **Show** that

$$D_n = \frac{4(1-\cos n\pi)}{\pi n^3}.$$

(5 marks)

(d) Hence, **show** that

$$u(x,t) = \frac{8}{\pi}\left\{\frac{(\sin x)e^{-3(1)^2 t}}{1^3} + \frac{(\sin 3x)e^{-3(3)^2 t}}{3^3} + \frac{(\sin 5x)e^{-3(5)^2 t}}{5^3} + \dots\right\}$$

(5 marks)

PART B

Q3 (a) Given the first order ordinary differential

$$(t-4)y^4 dt - t^3(y^2-3)dy = 0.$$

(i) Show that the differential equation is separable.

(ii) Hence, solve the equation.

(5 marks)

(b) (i) Show that the differential equation

$$x\frac{dy}{dx} + y = e^{2x}$$

is exact.

(ii) Hence, show that its solution is

$$2xy - e^{2x} = A,$$

where A is an arbitrary constant.

(7 marks)

(c) Solve the following initial value problem,

$$(\cos x)\frac{dy}{dx} + (\sin x)y = 2\cos^3 x\sin(x-1), \quad y\left(\frac{\pi}{4}\right) = 3\sqrt{2}, \quad 0 \le x < \frac{\pi}{2}.$$

(8 marks)

Q4 Given $y'' - 4y' + 4y = f(x)$.

(a) Solve for $f(x) = 0$.

(3 marks)

(b) Solve for $f(x) = (x+5)^2$ by using the method of undetermined coefficients.

(9 marks)

(c) Solve for $f(x) = \dfrac{x}{e^{-2x}}$ by using the method of variation of parameters.

(8 marks)

Q5 (a) Find $\mathcal{L}^{-1}\left\{\dfrac{s+3}{s^2+8s+12}\right\}$.

(5 marks)

(b) Consider the function

$$f(t)=\begin{cases} e^{3t} & 0\le t<1, \\ t-1 & t\ge 1. \end{cases}$$

(i) Write the function $f(x)$ in the form of unit step functions.

(ii) Then, find the Laplace transform of $f(t)$.

(6 marks)

(c) Solve $y''+4y=8\delta(t-2\pi)$, $y(0)=3$, $y'(0)=0$.

(9 marks)

Q6 Given

$$f(x)=\begin{cases} 2x & -3\le x<0, \\ -2x & 0\le x<3, \end{cases}$$
$$f(x)=f(x+6).$$

(a) Sketch the graph of $f(x)$ for $-10\le x\le 10$.

(3 marks)

(b) Determine whether $f(x)$ is an odd, even or neither function. Give your reason.

(2 marks)

(c) Find the Fourier coefficients; a_0, a_n, and b_n.

(9 marks)

(d) Obtain the Fourier series expansion of the periodic function $f(x)$.

(3 marks)

(e) Find the value for the series $1+\frac{1}{9}+\frac{1}{25}+\cdots$

(3 marks)

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay''+by'+cy=0$.

Characteristic equation: $am^2+bm+c=0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y=Ae^{m_1x}+Be^{m_2x}$
2.	Real and equal roots: $m=m_1=m_2$	$y=(A+Bx)e^{mx}$
3.	Complex roots: $m_1=\alpha+\beta i$, $m_2=\alpha-\beta i$	$y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay''+by'+cy=f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x)=A_nx^n+A_{n-1}x^{n-1}+\cdots+A_1x+A_0$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^re^{\alpha x}(P\cos\beta x+Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x\\ \sin\beta x\end{cases}$	$x^r(B_nx^n+B_{n-1}x^{n-1}+\cdots+B_1x+B_0)e^{\alpha x}\cos\beta x+$ $x^r(C_nx^n+C_{n-1}x^{n-1}+\cdots+C_1x+C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_h=Ay_1+By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y=uy_1+vy_2,$$

where $u=-\int\frac{y_2f(x)}{aW}dx+A$, $v=\int\frac{y_1f(x)}{aW}dx+B$ and $W=\begin{vmatrix}y_1 & y_2\\ y_1' & y_2'\end{vmatrix}=y_1y_2'-y_2y_1'$.

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Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
A	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

Fourier Series

Fourier series expansion of periodic function with period $2L$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

Fourier half-range series expansion

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$