UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	MATHEMATICS II
CODE	:	DSM 1923
COURSE	:	1 DDM / DDT / DFA / DFT 2 DFT
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 9 PAGES

PART A

Q1 (a) Determine if the improper integral, $\int_0^{\infty} \frac{dx}{x^2+1}$ converges or diverges. If it converges, determine its value.

(6 marks)

(b) Evaluate

(i) $\int_2^4 \frac{xdx}{\sqrt{x^2-4}}$.

(ii) $\int_1^2 \frac{dx}{1-x}$.

(14 marks)

Q2 (a) Refer to **Figure Q2(a)**. Find

(i) the coordinates of A and B.

(ii) the area of the shaded region.

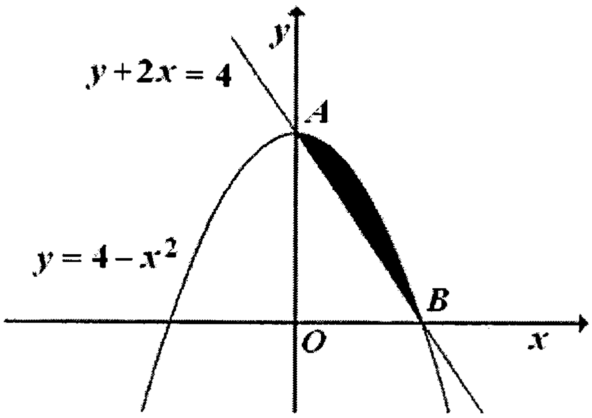

Figure Q2(a)

(6 marks)

(b) Refer to **Figure Q2(b).** Find

(i) the coordinate A.

(ii) the volume of solid revolution generated by rotating the shaded region about the x-axis.

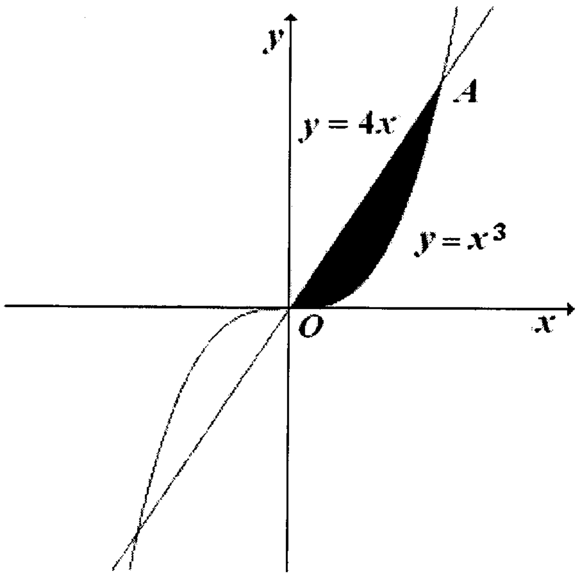

Figure Q2(b)

(7 marks)

(c) Use the Simpson's rule to approximate the integral of $\int_{2}^{6} \frac{x}{1+x^3}\,dx$ with subinterval, $h = 0.4$. Give the answer in four decimal places.

(7 marks)

PART B

Q3 (a) Find

(i) $\lim_{x \to +\infty} \dfrac{3x^2 - x}{2x^3 - 5}$.

(ii) $\lim_{x \to -\infty} \dfrac{3x^3 - 2x^2 + 1}{2x + 5}$.

(6 marks)

(b) Given a function of $f(x) = \begin{cases} \dfrac{1}{x+2}, & x < -2 \\ x^2 - 5, & -2 < x \leq 3 \\ \sqrt{x+13}, & x > 3 \end{cases}$.

Check whether f is continuous at $x = -2$, $x = 0$ and $x = 3$.

(6 marks)

(c) (i) A curve is given by a parametric equation $x = t^3$ and $y = 3 - 6t - 4t^3$.

Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of t.

(ii) By using the chain rule, find the derivative of $y = \ln(\sin x - \cos 2x)$.

(8 marks)

Q4 (a) Find $\dfrac{dy}{dx}$ of

(i) $y = \dfrac{e^{-2x}}{1+x^2}$.

(ii) $y = \cot^{-1}\sqrt{x}$.

(iii) $y = x^2(x^2-1)^3(2x+1)^4$.

(Hint: Use logarithm differentiation technique).

(13 marks)

(b) The drop in voltage V (in milivolt) when current is passing through a circuit is given by

$$V(t) = t^3 - 9t^2 + 24t,$$

where t is time (in second) and $0 \leq t \leq 6$. Find the maximum value of the drop in voltage.

(7 marks)

Q5 (a) Find the limits below by using L'Hôpital's rule.

(i) $\lim_{x\to\infty}\left(\frac{1}{x^2}\right)e^x$

(ii) $\lim_{x\to\infty}\left[\left(\frac{x}{2}\right)\cdot\ln\left(1+\frac{3}{x}\right)\right]$

(12 marks)

(b) Evaluate $\int_1^6 \frac{3x+7}{x^2-2x-3}\,dx$ by using partial fraction method.

(8 marks)

Q6 (a) Evaluate each integral using appropriate technique.

(i) $\int x^2 e^{-2x}\,dx$

(ii) $\int \frac{4x+6}{(x^2+3x+7)^4}\,dx$

(iii) $\int_0^5 \frac{x}{\sqrt{x+4}}\,dx$

(14 marks)

(b) Compute the arc length of $y=6(x-3)^{\frac{3}{2}}$ from $x=3$ to $x=10$. Do all calculation in three decimal places.
(Hint: Use substitution method in the integration process).

(6 marks)

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FORMULAE

DIFFERENTIATION

$\frac{d}{dx}[ax] = a$	$\frac{d}{dx}[\sin ax] = a\cos ax$
$\frac{d}{dx}[x^n] = nx^{n-1}$	$\frac{d}{dx}[\cos ax] = -a\sin ax$
$\frac{d}{dx}[c\,f(x)] = c\,\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\tan ax] = a\sec^2 ax$
$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$	$\frac{d}{dx}[\sec ax] = a\sec ax\tan ax$
$\frac{d}{dx}[f(x)g(x)] = f(x)\cdot\frac{d}{dx}[g(x)] + g(x)\cdot\frac{d}{dx}[f(x)]$	$\frac{d}{dx}[\cot ax] = -a\csc^2 ax$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\cdot\frac{d}{dx}[f(x)] - f(x)\cdot\frac{d}{dx}[g(x)]}{[g(x)]^2}$	$\frac{d}{dx}[\csc ax] = -a\csc ax\cot ax$
$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$	$\frac{d}{dx}\left[\sin^{-1} ax\right] = \frac{1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}\left[e^{ax}\right] = ae^{ax}$	$\frac{d}{dx}\left[\cos^{-1} ax\right] = \frac{-1}{\sqrt{1-a^2x^2}}\frac{d}{dx}[ax]$
$\frac{d}{dx}\left[a^x\right] = a^x \ln a$	$\frac{d}{dx}\left[\tan^{-1} ax\right] = \frac{1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}\ln\lvert ax+b\rvert = \frac{a}{ax+b}$	$\frac{d}{dx}\left[\cot^{-1} ax\right] = \frac{-1}{1+a^2x^2}\frac{d}{dx}[ax]$
$\frac{d}{dx}[\log_a u] = \frac{1}{u}\log_a e\,\frac{d}{dx}[u(x)]$	$\frac{d}{dx}\left[\sec^{-1} ax\right] = \frac{1}{\lvert ax\rvert\sqrt{a^2x^2-1}}\frac{d}{dx}[ax]$

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INTEGRATION

$$\int c\, f(x)dx = c\, F(x) + C$$

$$\int [f(x) \pm g(x)]\, dx = F(x) \pm G(x) + C$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C\,, (n \neq -1)$$

$$\int u\, dv = uv - \int v\, du$$

$$\int \frac{1}{ax+b}\, dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int \sin(ax)\, dx = -\frac{\cos(ax)}{a} + C$$

$$\int \cos(ax)\, dx = \frac{\sin(ax)}{a} + C$$

$$\int \tan(ax)\, dx = \ln|\sec(ax)| + C$$

$$\int \sec^2(ax)\, dx = \frac{\tan(ax)}{a} + C$$

$$\int \csc^2(ax)\, dx = -\frac{\cot(ax)}{a} + C$$

$$\int \sec(ax)\tan(ax)\, dx = \frac{\sec(ax)}{a} + C$$

$$\int \csc(ax)\cot(ax)\, dx = -\frac{\csc(ax)}{a} + C$$

$$\int \csc(ax)\, dx = -\ln|\csc(ax) + \cot(ax)| + C$$

$$\int \sec(ax)\, dx = \ln|\sec(ax) + \tan(ax)| + C$$

$$\int \frac{1}{\sqrt{a^2 - u^2}}\, du = \sin^{-1}\left(\frac{u}{a}\right) + C\ ;\ u^2 < a^2$$

$$\int \frac{1}{\sqrt{a^2 + u^2}}\, du = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\, du = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C\ ;\ u^2 < a^2$$

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IMPROPER INTEGRAL

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$$

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^0 f(x)\,dx + \int_0^\infty f(x)\,dx = \lim_{a\to-\infty}\int_a^0 f(x)\,dx + \lim_{b\to\infty}\int_0^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \lim_{m\to c^-}\int_a^m f(x)\,dx + \lim_{n\to c^+}\int_m^b f(x)\,dx$$

AREA OF REGION

$$A = \int_a^b [f(x)-g(x)]\,dx \quad \text{OR} \quad A = \int_c^d [w(y)-v(y)]\,dy$$

VOLUME OF REVOLUTION

$$V = \pi\int_a^b \left\{[f(x)]^2 - [g(x)]^2\right\} dx \quad \textbf{OR} \quad V = \pi\int_c^d \left\{[w(y)]^2 - [v(y)]^2\right\} dy$$

ARC LENGTH

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \textbf{OR} \quad L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

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SIMPSON'S RULE

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0 + f_n) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i\right] \quad ; \quad n = \frac{b-a}{h};\ x_i = a + ih$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i\right] \quad ; \quad n = \frac{b-a}{h};\ x_i = a + ih$$