# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

# FINAL EXAMINATION
# SEMESTER II
# SESSION 2008/2009

| | | |
|---|---|---|
| SUBJECT | : | MATHEMATICS II |
| CODE | : | DSM 1933 |
| COURSE | : | 1 DET / DEE |
| DATE | : | APRIL 2009 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B** |

THIS EXAMINATION PAPER CONSISTS OF 5 PAGES

**PART A**

**Q1** (a) Determine $\frac{dy}{dx}$ for the following functions.

(i) $y=\frac{e^{x^2}}{e^{x+1}}$

(ii) $y=\ln\left(\ln\left(\frac{1}{x}\right)\right)$

(iii) $y=\log_3\left(\frac{x^2}{4-x^2}\right)$; $\left(\text{Hint: } \frac{d}{dx}\log_a u=\frac{1}{u}\log_a e\ \frac{du}{dx}, u\neq 0\right)$

(iv) $y^4+2x^2y^2+6x^2=7$

(v) $x=te^{-t},\quad y=2t^2+1$

(16 marks)

(b) If the displacement (in metres) at time $t$ (in seconds) of an object is given by $s=4t^3+7t^2-2t$, find the acceleration at time $t=10$.

(4 marks)

**Q2** (a) Find the Laplace transform of the following functions.

(i) $f(t)=3e^{4t}-6+5\cos 2t$

(ii) $f(t)=t\sin 2t$

(5 marks)

(b) Find the inverse Laplace transform of the following functions.

(i) $F(s)=\frac{1}{s^2+s}$

(ii) $F(s)=\frac{2s}{(s-1)^2+4}$

(8 marks)

(c) Given

$$f(t)=\begin{cases} t, & 0<t\leq 2, \\ 4-t, & 2<t\leq 4, \\ 0, & t>4. \end{cases}$$

(i) Express $f(t)$ in the form of unit step function.

(ii) Find the Laplace transform of $f(t)$.

(7 marks)

**PART B**

**Q3** (a) Find the value(s) of $x$ and $y$ if $\begin{bmatrix} x & y-2 \\ y & 1 \end{bmatrix}\begin{bmatrix} 2 \\ x-1 \end{bmatrix}=\begin{bmatrix} 5 \\ 7 \end{bmatrix}$.

(6 marks)

(b) Solve the following system of linear equations by using Gauss-Jordan Elimination method.

$$-2x-y=3,$$
$$4x+11y=30.$$

(6 marks)

(c) Solve the system below by the Gauss-Seidel method. Use $X^{(0)}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ and $\varepsilon=0.01$.

$$\begin{bmatrix} 7 & 1 & -1 \\ 0 & 5 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 10 \\ -9 \\ 1 \end{bmatrix}$$

(8 marks)

**Q4** (a) Let $\mathbf{u}=4\mathbf{i}+\mathbf{k}$, $\mathbf{v}=2\mathbf{i}-\mathbf{j}$ and $\mathbf{w}=a\mathbf{i}+3\mathbf{j}-4b\mathbf{k}$. Find

(i) $4\mathbf{u}-3\mathbf{v}+\mathbf{w}$.

(ii) $\mathbf{u}\times\mathbf{w}$.

(iii) the value of $a$ and $b$ if $4\mathbf{u}-3\mathbf{v}+\mathbf{w}=\mathbf{u}\times\mathbf{w}$.

(iv) a unit vector, $\mathbf{y}$ with the same direction as $\mathbf{w}$.

(11 marks)

(b) Find the distance between planes $-2x+y+z=0$ and $6x-3y-3z-5=0$.

(4 marks)

(c) Find the parametric and symmetric equation that passes through points $P(-2,0,3)$ and $Q(3,5,-2)$.

(5 marks)

**Q5** (a) (i) Solve for $z$ if $(i-z)+(2z-3i)=-2+7i$.

(ii) Find the real numbers $a$ and $b$ that satisfy the equation

$a(2+3i)+b(1-4i)=7+5i$.

(6 marks)

(b) Given $z_1=1-2i,\ z_2=4+3i$ and $z_3=10i$, find $z_4$ if $\dfrac{z_1}{z_2}=\dfrac{z_3}{z_4}$.

(4 marks)

(c) By using De Moivre theorem,

(i) evaluate $z=(-4+4i)^3$. Express the answer in the form of $a+ib$.

(ii) find all the roots of $z=(-4+4i)^{1/5}$. Express the answer in the form of $a+ib$.

Then, sketch them in an Argand Diagram.

(10 marks)

**Q6** (a) Evaluate the following limits.

(i) $\displaystyle\lim_{x\to1}\frac{3(x-1)+4(x-1)^2}{2(x-1)+5(x-1)^2}$

(ii) $\displaystyle\lim_{x\to0}\frac{1-\frac{2}{x}}{x^2-4}$

(iii) $\displaystyle\lim_{x\to0}\frac{2-\sqrt{4+x}}{x}$

(iv) $\displaystyle\lim_{x\to\infty}\frac{4x^5-2x+3}{1-2x+x^4}$

(12 marks)

(b) If $\lim_{x\to c}f(x)=2$ and $\lim_{x\to c}g(x)=-3$, evaluate $\displaystyle\lim_{x\to c}\frac{[f(x)]^2}{1-g(x)}$.

(4 marks)

(c) Suppose that the function $f(x)$ is continuous at $x=5$ and that $f(x)$ is defined by the rule

$$f(x)=\begin{cases}kx^2+2 & \text{if } x<5\\ 4x+7 & \text{if } x\geq5\end{cases}.$$

Find

(i) $k$.

(ii) $\lim_{x\to5}f(x)$.

(4 marks)

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SEMESTER / SESSION: SEM II / 2008/2009 COURSE : 1 DET / DEE

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## FORMULAS

**Table 1: Laplace transforms.**

| $L\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$ | |
|---|---|
| $f(t)$ | $F(s)$ |
| $k$ | $\frac{k}{s}$ |
| $t^n,\ n=1,2,..$ | $\frac{n!}{s^{n+1}}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ |
| $e^{at}f(t)$ | $F(s-a)$ |
| $t^n f(t),\ n=1,2,..$ | $(-1)^n \frac{d^n F(s)}{ds^n}$ |
| $f(t-a)\,H(t-a)$ | $e^{-as}F(s)$ |

**Table 2: Differentiation**

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}\ln u = \frac{1}{u}\cdot\frac{du}{dx}$$

$$\frac{d}{dx}e^u = e^u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\sin u = \cos u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\cos u = -\sin u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u\tan u\cdot\frac{du}{dx}$$

$$\frac{d}{dx}\csc u = -\csc u\cot u\cdot\frac{du}{dx}$$