UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	MATHEMATICS III
CODE	:	DSM 2913
COURSE	:	2 DDT / DDM / DFT 3 DFA / DFX
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 6 PAGES

PART A

Q1 (a) For each of the following, find either $F(s)$ or $f(t)$ as indicated.

(i) $\mathcal{L}\left[t^2 - e^{2t} + 1\right]$

(ii) $\mathcal{L}\left[te^{-t} - t\cos t\right]$

(iii) $\mathcal{L}^{-1}\left[\frac{2}{s+6} - \frac{5}{s^3} + \frac{9}{s^2-9}\right]$

(iv) $\mathcal{L}^{-1}\left[\frac{1}{(s+1)(s+2)}\right]$

(14 marks)

(b) Consider the piecewise function

$$h(t) = \begin{cases} 1, & 0 \le t < 4, \\ 5-t, & 4 \le t < 5, \\ 0, & t \ge 5. \end{cases}$$

(i) Write the function $h(t)$ in the form of unit step function.
(ii) Find the Laplace transform of $h(t)$.

(6 marks)

Q2 (a) (i) By using the **Convolution Theorem**, find

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{(s^2+4)s}\right].$$

(ii) Evaluate $f\left(\frac{\pi}{4}\right)$ and $f(\pi)$.

(14 marks)

(b) Use the result in (a)(i) to find

$$\mathcal{L}^{-1}\left[\frac{s+e^s}{(s^2+4)s}\right].$$

(6 marks)

PART B

Q3 (a) Given

$$2x + y = 5,$$
$$2x + 4y + z = -2,$$
$$x + y + 3z = 7.$$

Find the solution of the linear system above by using **Gauss Seidel** method until $\max|\text{error}| \leq 0.0005$. Use $x^{(0)} = y^{(0)} = z^{(0)} = 0$ as initial starter and give your answer in 4 decimal points.

(6 marks)

(b) Given $A = \begin{pmatrix} 1 & 4 \\ 2 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 5 & -7 \\ -2 & \frac{3}{2} \end{pmatrix}$ and $C = \begin{pmatrix} 2 & 5 \\ 0 & -4 \end{pmatrix}$.

(i) Calculate $\dfrac{A^{-1} + BB^T}{|C|}$.

(ii) Show that $(AB)^T = B^T A^T$.

(14 marks)

Q4 (a) Find a unit vector, **u** which is parallel to the line below

$$\frac{x-4}{2} = \frac{2y+1}{-3} = \frac{z-3}{3}.$$

(5 marks)

(b) Given $\mathbf{u} = \langle 5, 6, -7 \rangle$ and $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j} + \mathbf{k}$. Find $(\mathbf{u} \times \mathbf{v}) \times (\mathbf{u} \times \mathbf{v})$. Show your work.

(5 marks)

(c) (i) Given $i = \sqrt{-1}$, simplify i^9.

(ii) Calculate $5[\cos 20^\circ + i \sin 20^\circ] \times 4[\cos 115^\circ - i \sin 115^\circ]$.
Write the answer in standard form, $a + ib$ in 4 decimal points.

(4 marks)

(d) Given $z = 2i$. Apply de Moivre's Theorem to evaluate $\sqrt[3]{z^4}$. Express the results in the polar form.

(6 marks)

Q5 (a) Given a first order differential equation

$$\frac{dy}{dx} = \frac{y+x}{x}.$$

(i) Show that the equation is a homogeneous equation.
(ii) Solve the equation.

(8 marks)

(b) A body at unknown temperature is placed in a room which is held at a constant temperature of 30°F. After 10 minutes, the temperature of the body is 0°F, and after 20 minutes the temperature of the body is 15°F. The Newton's Law of Cooling equation for this problem is

$$\frac{dT}{dt} - kT = -30k.$$

where T is the variable for temperature in °F, t is the variable for time in minutes and k is a constant of proportionality.

(i) By using the linear equation method, show that the general solution for the differential equation is

$$T(t) = 30 + ce^{kt}.$$

(ii) Then, find the value of c and k.
(iii) Hence, find the unknown initial temperature.

(12 marks)

Q6 (a) Solve $y'' - 2y' + y = \frac{e^x}{x}$ by using the method of variation of parameters.

(10 marks)

(b) Given the initial value problem as follows;

$$\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 2e^{3t}, \qquad y(0) = 5 \text{ and } y'(0) = 7.$$

(i) Show that the Laplace transform for the given problem is

$$Y(s) = \frac{1}{(s^2 - 3s + 2)}\left(\frac{2}{s-3} + 5s - 8\right).$$

(ii) Then, use partial fractions to prove that

$$\frac{1}{(s^2 - 3s + 2)}\left(\frac{2}{s-3} + 5s - 8\right) = \frac{1}{s-3} + \frac{4}{s-1}.$$

(iii) Finally, solve the given problem.

(10 marks)

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$
$P_n(x)\, e^{\alpha x}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\cdot\begin{cases}\cos\beta x \\ \text{or} \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0{}^n)\sin\beta x$
$Ce^{\alpha x}\cdot\begin{cases}\cos\beta x \\ \text{or} \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(p\cos\beta x + q\sin\beta x)$
$P_n(x)e^{\alpha x}\cdot\begin{cases}\cos\beta x \\ \text{or} \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x$ + $x^r(C_n x^n + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$
a	$\dfrac{a}{s}$
$t^n,\ n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n = 1, 2, 3, \ldots$	$(-1)^n \dfrac{d^n F(s)}{ds^n}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s) - y(0)$
$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$