UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2008/2009

SUBJECT	:	MATHEMATICS III
CODE	:	DSM 2933
COURSE	:	2 DEE / DET / DEX
DATE	:	APRIL 2009
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER ALL QUESTIONS IN **PART A** AND CHOOSE **THREE (3)** QUESTIONS ONLY IN **PART B**

THIS EXAMINATION PAPER CONSISTS OF 7 PAGES

PART A

Q1 (a) Find the solution of the differential equation

$$\left(y^2 + xy^2\right)dx - \left(x^2 y - x^2\right)dy = 0$$

which satisfies $y(2) = 1$.

(8 marks)

(b) Given that

$$\frac{dy}{dx} = \frac{3x^2 + y}{3y^2 - x}.$$

(i) Show that the differential equation above is an exact equation.

(ii) Then, solve the equation.

(12 marks)

Q2 (a) Solve $y'' - 6y' + 10y = 0$.

(5 marks)

(b) Find the solution of the equation below.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 12e^x,\ y(0) = 0,\ y'(0) = 2$$

(15 marks)

PART B

Q3 (a) Suppose a forest fire spreads in a circle with radius changing at a rate of 5 feet per minute. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

(5 marks)

(b) Let $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2-6x+8$.

(i) Find the intervals on which $f(x)$ is increasing and the intervals on which it is decreasing.

(ii) Identify the locations of the relative extrema and inflection point for the function $f(x)$. Hence sketch its graph.

(15 marks)

Q4 Evaluate

(a) $\int x^2 \sin 3x \, dx$.

(8 marks)

(b) $\int (4x+6)\sqrt{x^2+3x} \, dx$.

(4 marks)

(c) $\int \frac{3x^2-2x+12}{(x^2+3)(x+2)} \, dx$.

(8 marks)

Q5 (a) Evaluate $\int_{-1}^{0} \frac{y+47}{95y-34}\,dy$ by using the Simpson's rule with 8 subintervals.

(6 marks)

(b) Find the area of the region enclosed by $y = 3 - x$ and $y = x^2 - 9$.

(8 marks)

(c) Find the volume of the solid that is obtained when the shaded region R in Figure Q5 is revolved about the $y-$axis.

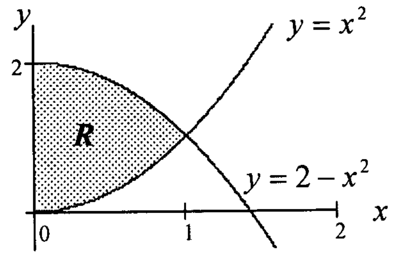

Figure Q5

(6 marks)

Q6 (a) Solve the homogeneous equation $x\,dy - y\,dx - \sqrt{x^2 - y^2}\,dx = 0$.

(10 marks)

(b) Given

$$\left(6x^2 - 10xy + 3y^2\right)dx + \left(-5x^2 + 6xy - 3y^2\right)dy = 0.$$

(i) Show that the given equation is an exact equation.

(ii) Then, solve the differential equation above.

(10 marks)

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Formulae

Differentiation And Integration Formula

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x \rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\ dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\ dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\ dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\ dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\ dx = \sec x + C$
$\frac{d}{dx}\csc x = -\csc x \cot x$	$\int \csc x \cot x\ dx = -\csc x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C$
$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$

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Volume, Arc Length and Surface Area of Revolution

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

$$V = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2\right) dx$$

$$V = \pi \int_c^d [w(y)]^2 \, dy$$

$$V = \pi \int_c^d \left([w(y)]^2 - [v(y)]^2\right) dy$$

$$V = 2\pi \int_a^b x\, f(x) \, dx$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

$$S = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$S = 2\pi \int_a^b g(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

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Numerical Integration

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{i=1}^{n-1} f(x_i)\right]$$

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a)+f(b)+4\sum_{\substack{i=1\\ i\ odd}}^{n-1} f(a+ih)+2\sum_{\substack{i=2\\ i\ even}}^{n-2} f(a+ih)\right]$$

Particular Integral of $ay''+by'+cy=f(x)$

$f(x)$	$y_p(x)$
$P_n(x)=A_0+A_1x+\cdots+A_nx^n$	$x^r(B_0+B_1x+\cdots+B_nx^n)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x+q\sin\beta x)$

Note : r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

Variation of Parameters Method for $ay''+by'+cy=f(x)$

$$y(x)=uy_1+vy_2$$

$$u=-\int\frac{y_2 f(x)}{aW}\,dx+A \qquad v=\int\frac{y_1 f(x)}{aW}\,dx+B$$

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$