Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME	:	2 DAU
EXAMINATION DATE	:	JUNE/JULY 2016
DURATION	:	3 HOURS
INSTRUCTIONS	:	SECTION A) ANSWER **ALL** QUESTIONS SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 Integrate the following:

(a) $\int\left(x^3+2\sqrt{x}-\frac{3}{x^2}\right)dx$

(5 marks)

(b) $\int x^2\left(x-5x^2+\frac{2}{x^{-3}}\right)dx$

(5 marks)

(c) $\int\left(\frac{7x^{-2}+x^4+x-6}{x}\right)dx$

(5 marks)

(d) $\int\left(\sin 2x+e^{3x}-\frac{4}{x+1}\right)dx$

(5 marks)

Q2 (a) Given two curves, $y=-x^2+6$ and $y=-x$,

(i) Sketch the region that enclosed by both curves. Include all intersection points.

(5 marks)

(ii) Hence, calculate the area of the bounded region.

(7 marks)

(b) Find the volume of the solid generated when the region enclosed between curves $y=x^3$ and $y=\sqrt{x}$, which is revolved about the y-axis.

(8 marks)

PART B

Q3 Sketch the graph and determine the domain and range.

(a) $y = x^2 - 5$

(5 marks)

(b) $y = \sqrt{x-4}$

(5 marks)

(c) $y = -\dfrac{1}{(x+4)^2}$

(5 marks)

(d) $y = \begin{cases} x^3 + 1 & , \quad x \geq 0 \\ -x + 2 & , \quad x < 0 \end{cases}$

(5 marks)

Q4 (a) By referring to **Figure Q4**,

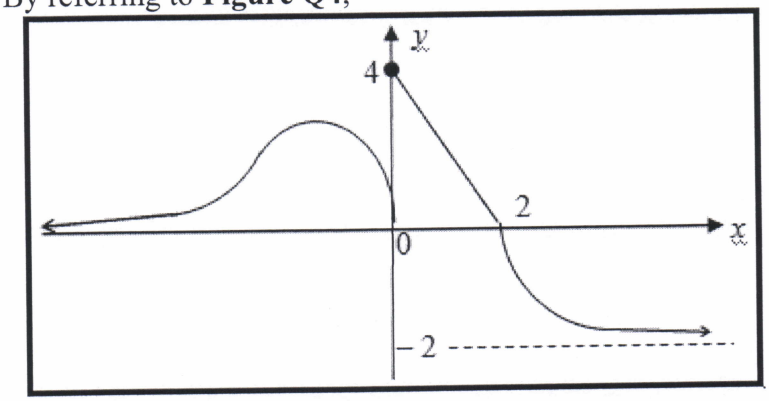

Figure Q4

(i) Find $\lim_{x \to 0^+} f(x)$, $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0} f(x)$.

(3 marks)

(ii) Find $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2} f(x)$. Verify whether $f(x)$ continuous at $x = 2$ or not.

(5 marks)

(iii) Find $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

(2 marks)

(b) Evaluate the limits of the following expressions.

(i) $\lim_{x \to 1} \dfrac{x^3 - 5x^2 + 1}{x + 1}$

(2 marks)

(ii) $\lim_{x \to -1} \dfrac{x^2 - 2x - 3}{x + 1}$

(4 marks)

(iii) $\lim_{x \to 0} \dfrac{1 - \sqrt{1 + x}}{x}$

(4 marks)

Q5 Differentiate the following:

(a) $y = -2x^4 + 3x - 7 - \dfrac{1}{x} + 4\sqrt{x}$

(3 marks)

(b) $y = \sin\left(e^{3x^2}\right)$

(3 marks)

(c) $3x^2 + y^3 = 4$ by using implicit differentiation.

(4 marks)

(d) $y = \left(x^4 + x^2 + 6x - \dfrac{1}{x^2}\right)(x^3 - 3x + 4)$ by using product rule.

(5 marks)

(e) $y = \dfrac{\ln x + \cos x}{x^2 - 4x}$ by using quotient rule.

(5 marks)

Q6 (b) By using L'Hospital's Rule, find

(i) $\lim_{x\to 0}\dfrac{2x^2-x}{2x^3}$

(2 marks)

(ii) $\lim_{x\to \infty}\dfrac{\ln(x-1)}{x}$

(3 marks)

(iii) $\lim_{x\to \infty}\dfrac{3e^x}{x^2}$

(3 marks)

(b) Given a curve $f(x)=\dfrac{1}{3}x^3+2x^2-5x+1$

(i) Find the critical points and inflection point.

(4 marks)

(ii) Fill up the **Table Q6**.

(5 marks)

Table Q6

Value Type	Test Value	Critical Value	Test Value	Inflection point	Test value	Critical Value	Test Value
x	$x=$	$x=$	$x=$	$x=$	$x=$	$x=$	$x=$
$f(x)$							
$f'(x)$							
$f''(x)$							
Graph Characteristics							

(iii) State the minimum/maximum point and inflection points if exist.

(3 marks)

Q7 Evaluate the following integrals

(a) $\int \frac{5}{x^2+5x-14}dx$ by using partial fractions.

(5 marks)

(b) $\int \frac{x+2}{\frac{x^2}{2}+2x-7}dx$ by using substitution.

(5 marks)

(c) $\int 2x\sqrt{1-2x}dx$ by using substituting $u^2 = 1-2x$.

(5 marks)

(d) $\int x^2 \ln x \, dx$ using integration by parts.

(5 marks)

– END OF QUESTION –

FINAL EXAMINATION

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Formula

Integration

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int e^x\,dx = e^x + C,$$

$$\int \frac{dx}{x} = \ln|x| + C,$$

$$\int \cos x\,dx = \sin x + C,$$

$$\int \sin x\,dx = -\cos x + C$$

Integration by part

$$\int uv' = uv - \int vu'$$

Application of Integration

$$A = \int_a^b [f(x) - g(x)]\,dx \quad \text{and} \quad V = \int_a^b 2\pi x\, f(x)\,dx$$

Differentiation

$$\frac{d}{dx}\left[x^n\right] = nx^{n-1}$$

$$\frac{d}{dx}\left[\tan x\right] = \sec^2 x$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

$$\frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x}$$

$$\frac{d}{dx}\left[\cos x\right] = -\sin x$$

$$\frac{d}{dx}\left[\sin x\right] = \cos x$$

Differentiation - Product Rule : $\frac{d}{dx}\left[u.v\right] = u\,v' + v\,u'$

Differentiation – Quotient Rule : $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\,u' - u\,v'}{v^2}$

Differentiation – Chain Rule : $(f \cdot g)'(x) = f'((g(x))g'(x)$