Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2015/2016

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAS 20603
PROGRAMME	:	3 DAE
EXAMINATION DATE	:	JUNE / JULY 2016
DURATION	:	3 HOURS
INSTRUCTION	:	SECTION A) ANSWER **ALL** QUESTIONS SECTION B) ANSWER **THREE (3)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

SECTION A

Q1 (a) Solve the initial value problem of the second order homogeneous differential equation below.

$$y'' + 25y = 0, \qquad y(0) = 3, \qquad y'(\pi) = 2$$

(9 marks)

(b) Find the general solution of the following second order non-homogeneous differential equation below.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 5e^{2x}$$

(11 marks)

Q2 Using method of undetermined coefficients, find the general solution of the given second order differential equations.

(a) $y''+9y=4x^2+2e^{3x}$

(11 marks)

(b) $\dfrac{d^2y}{dx^2}-8\dfrac{dy}{dx}+16y=3e^{3x}$

(9 marks)

SECTION B

Q3 (a) By using substitution technique, evaluate

$$\int_1^2\left(\frac{2x}{(x^2+3)^3}\right)dx.$$

(7 marks)

(b) Evaluate $\int 4x^3e^{2x}\,dx$ by using tabular method.

(7 marks)

(c) Solve $\int_1^2 \frac{x}{e^{2x}}dx$ by using Trapezoidal's rule, using $h=0.125$. Write the answer to 3 decimal places.

(6 marks)

Q4 (a) Show that the area of the region bounded by the curve $y = 4 - x^2$ and the line $y = 4 - 2x$ is $1\frac{1}{3}$ unit^2.

(9 marks)

(b) **Figure Q4(b)** shows the curve $y = \frac{2}{9}x^2$ and the line $y = 5 - x$. Determine

(i) coordinates of A and B.

(3 marks)

(ii) volume of the solid generated when the bounded region revolves 360° about y – axis using cylindrical shells.

(8 marks)

Q5 Given the first order differential equation

$$f(x,y) = \frac{x(\ln y + \ln x)}{y} \text{ and } g(x,y) = \frac{x^2 + y^2}{(y-x)(x+y)}.$$

(a) Determine whether the equations above are homogeneous equation or not.

(6 marks)

(b) Solve the homogeneous equation.

(14 marks)

Q6 The temperature of a dead old man when it was found at 5:00 am is 34°C. The surrounding temperature has been kept at a constant 29°C. After two hours, the temperature of the body is taken once more and found to be 30°C. Assuming that the victim's body temperature was normal (36°C) prior to death.

(a) By following the Newton's Law of Cooling,

$$\frac{dT}{dt} = -k(T - T_S)$$

Where T is temperature of the body, T_S is temperature surrounding body and k is the constant proportionality. Show that cooling equation can be written as $T = (T_0 - T_S)e^{-kt} + T_S$ if the initial condition $T = T_0$.

(10 marks)

(b) Determine the time of the death.

(10 marks)

Q7 (a) Given the differential equation of growth of decay problem can be formulated as

$$\frac{dN}{dt} = kN$$

Where $N(t)$ is the amount of material present and k is a constant proportionality. Using separable method, solve for $N(t)$.

(4 marks)

(b) Carbon-14 has a half life of 5730 years.

(i) If the initial amount of carbon-14 is 3000 gram, find how much is left after 2000 years.

(7 marks)

(ii) Determine when it will be if 350 grams carbon-14 left.

(3 marks)

(iii) Find how much it is left after 2000 years if the half life duration change to 3000 years.

(6 marks)

- END OF QUESTION –

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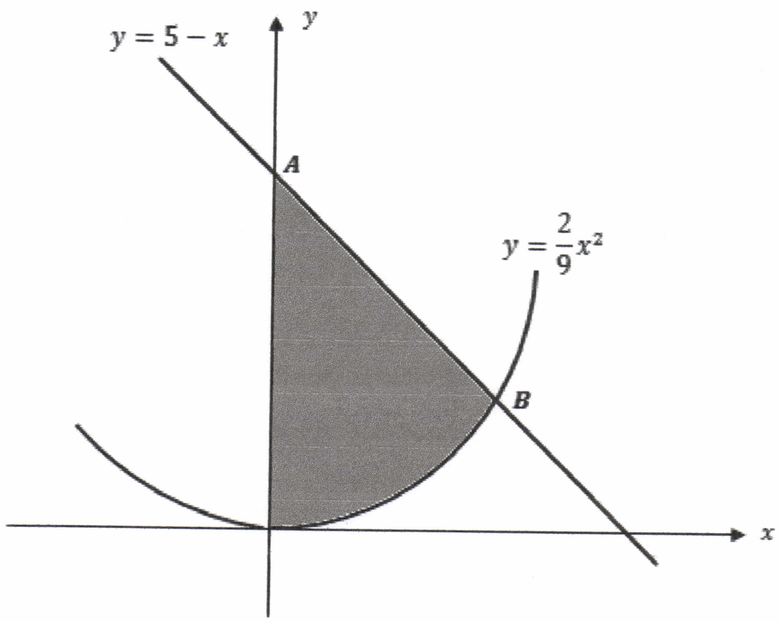

Figure Q4 (b)

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Formulae

Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$;
Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Method of Variation of Parameters

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx$ $\qquad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx$

Particular solution, $y_p = u_1 y_1 + u_2 y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$

Method Of Undetermined Coefficients

Case	$F(x)$	$y_p(x)$
1	Simple polynomial: $A_0 + A_1 x + \ldots + A_n x^n$	$x^r(B_0 + B_1 x + \ldots + B_n x^n)$, $r = 0, 1, 2,$
2	Exponential function: $Ce^{\alpha x}$	$x^r(ke^{\alpha x})$, $r = 0, 1, 2,$
3	Simple trigonometry: $C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$, $r = 0, 1, 2,$

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Trigonometry

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x\sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x\cos y = \cos(x+y) + \cos(x-y)$$

Differentiation and Integration

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\lvert x\rvert + C$
$\frac{d}{dx}\ln\lvert ax+b\rvert = \frac{1}{ax+b}$	$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin ax = a\cos ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\frac{d}{dx}\cos ax = -a\sin ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$

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Definite Integration

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Length of Curve

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$= \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$

Area

$$A = \int_a^b [f(x) - g(x)]\,dx$$
$$= \int_c^d [f(y) - g(y)]\,dy$$

Volume Cylindrical Method

$$V = 2\pi \int_a^b x\,f(x)\,dx$$
$$= 2\pi \int_c^d y\,f(y)\,dy$$

Area of Surface Revolution

$$S = 2\pi \int_a^b y(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$
$$= 2\pi \int_c^d x(y)\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$$