Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION (TAKE HOME) SEMESTER I SESI 2020/2021

COURSE NAME	:	ADVANCED DIGITAL SIGNAL PROCESSING
COURSE CODE	:	MEE 10303
PROGRAMME	:	MEE
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	6 HOURS
INSTRUCTION	:	ANSWER **FOUR** QUESTIONS ONLY **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) UMobile has developed a new Asymetric Digital Subscriber Line (ADSL) modem consist of the following system in **Figure Q1 (a)** with $x(n) = r(n+3) - r(n+2) + u(n+1) + \partial(n + Last\ Student\ id)$ $-2u(n-1)$ and the filter impulse response is given as $h_1(n) = r(n+1) - r(n)u(n-1) + \partial(n - Second\ Last\ Student\ id)$ and $h_2(n) = u(n+3) - u(n+2) - 2\partial(n-1)$. Find the output signal of the system.

(15 marks)

(b) Consider a damping system with the following difference equation :

$$y(n) = \frac{5}{6}y(n - Last\ student\ id) - \frac{1}{6}y(n-2) + x(n)$$

(i) Calculate the corresponding system function H(z).

(ii) Calculate the corresponding impulse response h(n).

(iii) Determine the pole-zero plot of the system. Is the system stable?

(10 marks)

Q2 (a) Assume that x[n] represents a right-sided signal. Determine the Region of Convergene (ROC) of the following z-transforms and compute x[n] using partial fractions.

(i) $$X(z) = \frac{Second\ Last\ Student\ id(z^2)}{(z^2-1.5z+0.5)(z-0.25)}$$

(ii) $$X(z) = \frac{4z}{(z+1)^2(z+Second\ Last\ Student\ id)}$$

(10 marks)

(b) Matsushita Sdn Bhd decided to develop a product twin-T notch filter having the following transfer function, $H(s)=\frac{s^2+1}{s^2+4s+1}$. The notch filter frequency, ω_0 is around 1 rad/s. Design the digital notch filter with the following parameter, where $S = 240$ Hz and a notch frequency $f = 60$ Hz.

(15 marks)

Q3 (a) Suppose Bartlett's and Welch's method is used to estimate the power spectrum of a signal $x(n)$ consisting of N = (First Student id)(Last Student id)00 samples.

(i) What is the minimum FFT block length (M) required to obtain a relative variance of 0.01 in your spectrum estimate, for Bartlett method and Welch method (using 50% overlap and hanning window)?

(ii) Determine the frequency resolution (Δf) of the spectrum estimate for Bartlett method and Welch method (using 50% overlap and hanning window).

(10 marks)

(b) In a Maxis Sdn Berhad company binary communication system (see **Figure Q3**), a 0 or 1 is transmitted. During transmission, due to channel noise, a 0 can be received as a 1 and vice versa. Let m_0 and m_1 denote the events of transmitting 0 and 1, respectively. Let r_0 and r_1 denote the events of receiving 0 and 1, respectively. Let $P(m_0)$ = 0.(Second student id), $P(r_1|m_0) = p$ = 0.(Second Last student id) and $P(r_0|m_1) = q = 0.2$.

(i) Predict the value for $P(r_0)$ and $P(r_1)$.

(ii) Deduce what is the probability that a 1 was sent, if a 1 was received?

(iii) Deduce what is the probability that a 0 was sent, if a 0 was received?

(iv) For the overall system, calculate the probability that the transmitted signal is correctly read at the receiver. Then estimate what is the probability of error P_e for the system.

(15 marks)

Q4 (a) The arrival time of a professor to his office is a continuous random variable uniformly distributed over the hour between 8 am and 9 am. Define the events:

A = {The prof. has not arrived by 8:30 am }

B = {The prof. will arrive by 8:31 am }

Find :

(i) $P(B|A)$.

(ii) $P(A|B)$.

(7 marks)

(b) A random variable X has a probability density :

$$f_X(x) = \begin{cases} \frac{\pi}{16}\cos\left(\frac{\pi x}{8}\right) & -4 \leq x \leq 4 \\ 0 & otherwise \end{cases}$$

Find :-

(i) its mean value $\bar{X}$.

(ii) its second moment $\overline{X^2}$.

(iii) its variance.

(9 marks)

(c) A random variable X has a probability density :

$$f_X(x) = \begin{cases} \frac{5}{4}(1 - x^4) & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Find : -

(i) E[X].

(ii) E[4X +2].

(iii) E[X^2].

(9 marks)

- END OF QUESTIONS –

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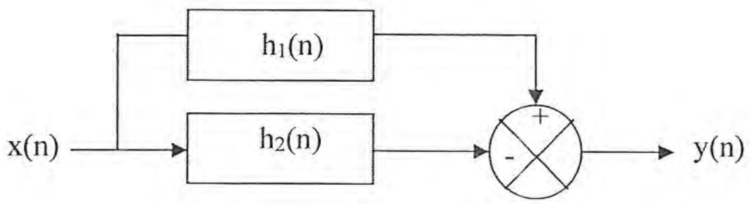

Figure Q1(a) : An ADSL modem system

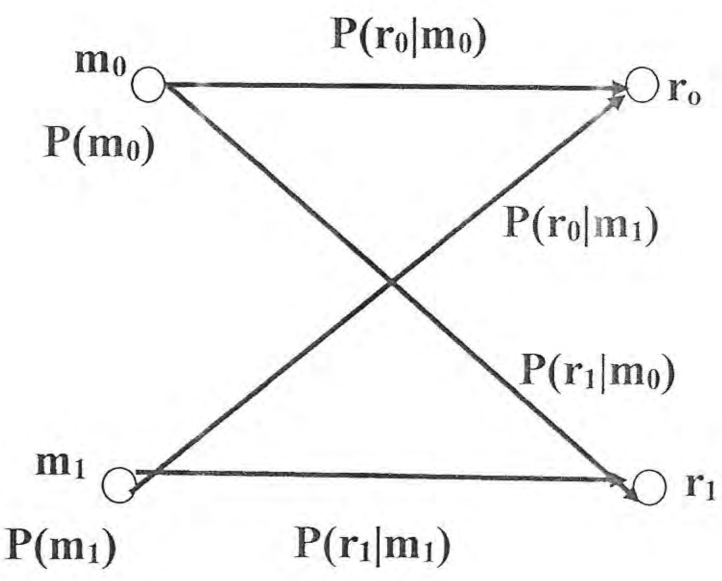

Figure Q3 : A binary communication system

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Appendix 1 : DTFT Properties

Sequence $x(n)$ $y(n)$	Fourier Transform $X(e^{j2\pi f})$ $Y(e^{j2\pi f})$
1. $ax(n)+by(n)$	$aX(e^{j2\pi f})+bY(e^{j2\pi f})$
2. $x(n-n_d)$, (n_d an integer)	$e^{-j2\pi f n_d}X(e^{j2\pi f})$
3. $e^{j2\pi f_0 n}x(n)$	$X(e^{j2\pi(f-f_0)})$
4. $x(-n)$	$X(e^{-j2\pi f})$ $X^*(e^{j2\pi f})$ if $x[n]$ real.
5. $nx(n)$	$j\dfrac{dX(e^{j2\pi f})}{d2\pi f}$
6. $x(n) * y(n)$	$X(e^{j2\pi f})Y(e^{j2\pi f})$
7. $x(n)\,y(n)$	$\int_{-f_s/2}^{f_s/2} X(e^{j2\pi v})Y(e^{j2\pi(f-v)})dv$
Parseval's Theorem	
8. $\frac{1}{N}\sum_{n=0}^{\infty}\lvert x(n)\rvert^2=\frac{1}{Nf_s}\int_{-f_s/2}^{f_s/2}\lvert X(e^{j2\pi f})\rvert^2 df$	
9. $\frac{1}{N}\sum_{n=0}^{N-1}x(n)y^*(n)=\frac{1}{Nf_s}\int_{-f_s/2}^{f_s/2}X(e^{j2\pi f})Y^*(e^{j2\pi f})df$	

Appendix 2 : Z-Transform Properties

Properties	$x(n)$	$X(z)$
Linearity	$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(z)+a_2X_2(z)$
Shift in time	$x(n+1)$	$z[X(z)-x(0)]$
Multiplication by n	$nx(n)$	$-z\frac{d}{dz}X(z)$
Multiplication by r^n	$r^n x(n)$	$X(\frac{z}{r})$
Convolution	$\sum_{k=0}^{\infty}x_1(k)x_2(n-k)$	$X_1(z)X_2(z)$
Initial Value	$x(0)$	$\lim_{z\to\infty}X(z)$
Final Value	$\lim_{n\to\infty}x(n)$	$\lim_{z\to 1}[(z-1)X(z)]$

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Appendix 3 : Z-Transform Pairs

$x(n)$ for $n \geq 0$	$X(z)$	Radius ROC $\|z\| > R$
$\delta(n)$	1	0
$\delta(n-m)$	z^{-m}	0
$u(n)$	$\frac{z}{z-1}$	1
n	$\frac{z}{(z-1)^2}$	1
n^2	$\frac{z(z+1)}{(z-1)^3}$	1
a^n	$\frac{z}{z-a}$	$\|a\|$
na^n	$\frac{az}{(z-a)^2}$	$\|a\|$
$(n+1)a^n$	$\frac{z^2}{(z-a)^2}$	$\|a\|$
$\frac{(n+1)(n+2)...(n+m)a^n}{m!}$	$\frac{z^{m+1}}{(z-a)^{m+1}}$	$\|a\|$
$\cos\Omega_0 n$	$\frac{z(z-\cos\Omega_0)}{z^2-2z\cos\Omega_0+1}$	1
$\sin\Omega_0 n$	$\frac{z\sin\Omega_0}{z^2-2z\cos\Omega_0+1}$	1
$a^n\cos\Omega_0 n$	$\frac{z(z-a\cos\Omega_0)}{z^2-2za\cos\Omega_0+a^2}$	$\|a\|$
$a^n\sin\Omega_0 n$	$\frac{za\sin\Omega_0}{z^2-2za\cos\Omega_0+a^2}$	$\|a\|$
$\exp[-anT]$	$\frac{z}{z-\exp[-aT]}$	$\|\exp[-aT]\|$
nT	$\frac{Tz}{(z-1)^2}$	1
$nT\exp[-anT]$	$\frac{Tz\exp[-aT]}{[z-\exp[-aT]]^2}$	$\|\exp[-aT]\|$
$\cos n\omega_0 T$	$\frac{z(z-\cos\omega_0 T)}{z^2-2z\cos\omega_0 T+1}$	1
$\sin n\omega_0 T$	$\frac{z\sin\omega_0 T}{z^2-2z\cos\omega_0 T+1}$	1
$\exp[-anT]\cos n\omega_0 T$	$\frac{z(z-\exp[-aT]\cos\omega_0 T)}{z^2-2z\exp[-aT]\cos\omega_0 T+\exp[-2aT]}$	$\|\exp[-aT]\|$
$\exp[-anT]\sin n\omega_0 T$	$\frac{z(z-\exp[-aT]\sin\omega_0 T)}{z^2-2z\exp[-aT]\cos\omega_0 T+\exp[-2aT]}$	$\|\exp[-aT]\|$

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Appendix 4 : DFT Properties

Properties	$x(n)$	$X(k)$
Linearity	$a_1x_1(n)+a_2x_2(n)$	$a_1X_1(k)+a_2X_2(k)$
Shift in time	$x(n-m)$	$e^{-j\frac{2\pi km}{N}}X(k)$
Shift in frequency	$e^{-j\frac{2\pi nm}{N}}x(n)$	$X(k-m)$
Circular convolution	$\sum_{\lambda=0}^{N-1}x_1(\lambda)x_2[(n-\lambda)\bmod N]$	$X_1(k)X_2(k)$
Multiplication	$x_1(n)x_2(n)$	$\sum_{\lambda=0}^{N-1}X_1(\lambda)X_2[(k-\lambda)\bmod N]$
Parseval's theorem	$N\sum_{n=0}^{N-1}\lvert x(n)\rvert^2$	$\sum_{k=0}^{N-1}\lvert X(k)\rvert^2$

The analog to digital transformation LP to BP and LP to BS for Notch and Peaking Filter

$$H_{BP}(z)=\frac{C}{1+C}\left(\frac{z^2-1}{z^2-\frac{2\beta}{1+C}z+\frac{1-C}{1+C}}\right)$$

$\beta=\cos\Omega_0$

$C=\tan(0.5\Delta\Omega)$

$\Delta\Omega=\Omega_2-\Omega_1$

$$H_{BS}(z)=\frac{1}{1+C}\left(\frac{z^2-2\beta z+1}{z^2-\frac{2\beta}{1+C}z+\frac{1-C}{1+C}}\right)$$

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Appendix 5 : Digital-to-digital frequency transformations

Form	Band Edges	Mapping $z \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-\alpha}{1-\alpha z}$	$\alpha = \dfrac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\dfrac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \dfrac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\dfrac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \dfrac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha K}{K+1}, \; A_2 = \dfrac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\dfrac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \dfrac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \dfrac{2\alpha}{K+1}, \; A_2 = \dfrac{1-K}{1+K}$

Appendix 6 : Direct analog-to-digital transformations for bilinear design

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\dfrac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\dfrac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{z^2 - 2\beta z + 1}{C(z^2 - 1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\dfrac{C(z^2 - 1)}{z^2 - 2\beta z + 1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \dfrac{\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$