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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	BDA 14103
PROGRAMME	:	BDD
EXAMINATION DATE	:	JULY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	PART A: ANSWER **ALL** QUESTIONS. PART B: ANSWER **THREE (3)** QUESTIONS **ONLY** OUT OF FOUR.

THIS QUESTION PAPER CONSISTS OF NINE **(9)** PAGES

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PART A

Q1 (a) The heat flux through the faces at the ends of bar is found to be proportional to $u_n = \partial u/\partial n$ at the ends. If the bar is perfectly insulated, also at the ends $x = 0$ and $x = L$ are adiabatic conditions,

$$u_x(0,t) = 0 \qquad u_x(L,t) = 0$$

prove that the solution of the heat transfer problem above (adiabatic conditions at both ends) gives as,

$$u(x,t) = A_o + \sum_{n=1} A_n \cos\frac{n\pi x}{L} e^{-\left(\frac{\alpha n\pi}{L}\right)^2 t}$$

where A_o and A_n are an arbitrary constant.

The heat equation given as,

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

(16 marks)

(b) If $L = \pi$ and $\alpha = 1$ for the solution of heat transfer problem in **Q1 (a)**, find the temperature in the bar with the initial temperature, $f(x) = k =$ constant.

(4 marks)

Q2 A half-range expansions given as the following function:

$$f(x)=\begin{cases} \frac{2k}{L}x & \text{for} \quad 0<x<\frac{L}{2} \\ \frac{2k}{L}(L-x) & \text{for} \quad \frac{L}{2}<x<L \end{cases}$$

(a) Sketch a graph of $f(x)$ in the interval $0<x<L$.

(2 marks)

(b) Solve the given half-range expansion if the function $f(x)$ is extended to the interval $-L<x<L$ as an *even function* and sketch the periodic extension for the series.

(12 marks)

(c) Solve the given half-range expansion if the function $f(x)$ is extended to the interval $-L<x<L$ as an *odd function* and sketch the periodic extension for the Fourier series.

(6 marks)

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PART B

Q3 (a) The given differential equation is not exact. Determine the integrating factor so that the given differential equation becomes exact. Hence, apply the integrating factor to solve the solution for the original equation.

$$\left(3x^2y+2xy+y^3\right)dx+\left(x^2+y^2\right)dy=0$$

(10 marks)

(b) Newton's Law of Cooling states that,

$$\frac{dT}{dt}=-k\left(T-T_s\right)$$

where,

T = object temperature
T_s = surrounding temperature
k = constant of proportionality

Let a metal is heated up to a temperature of 500°C, then exposed to a temperature of 38°C. After 2 minutes, the temperature of the metal becomes 190°C.

(i) What you can point out about the temperature of the metal after 4 minutes?

(ii) How long it takes time to the temperature of the metal dropped to 100°C?

(10 marks)

Q4 (a) Sketch the graph and express the following function in term of unit step functions.

$$f(t)=\begin{cases} t & 0\le t<2 \\ 4-t & 2\le t<4 \\ 0 & t\ge 4 \end{cases}$$

(6 marks)

(b) By using the relation obtained in **Q4 (a)**, solve the Laplace transforms of the function by using a second shift property.

(5 marks)

(c) Find the inverse Laplace transforms of the following expressions by using partial fraction.

$$\frac{3}{(s+2)(s-1)}$$

(9 marks)

Q5 (a) Identify whether or not the following equations are linear. Give your justification for the answers.

(i) $\left(x^2-1\right)\frac{dy}{dx}=x\left(y+\cos^{-1}x\right)$

(ii) $x^2\frac{dy}{dx}+y^2=y\sin x$

(4 marks)

(b) By using the method of variation of parameter, solve the following second order nonhomogeneous differential equation.

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=3e^x$$

(10 marks)

(c) If $\mathrm{L}\{f(t)\}=F(s-a)$ and a is a constant, demonstrate that the First Shift Theorem can be written as,

$$\mathrm{L}\left\{e^{at}f(t)\right\}=F(s-a)$$

(6 marks)

Q6 (a) Find the particular solution of the following differential equation that satisfies the given initial condition.

$$2y'' - 8y + 8y = 0 \qquad y(0) = 4 \quad y'(0) = 5$$

(10 marks)

(b) By using the method of undetermined coefficient, obtain the general solution for the following differential equation.

$$\frac{d^2y}{dx^2} + y = 3 - 2x^2$$

(10 marks)

- END OF QUESTION -

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SEMESTER / SESSION	: SEM 2 /20192020	PROGRAMME	: BDD
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FORMULAS

First Order Differential Equation

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx} Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\} dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int \left\{ \frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\} dy = C$

Characteristic Equation and General Solution for Second Order Differential Equation

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficient

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$	$x^r(A_n x^n + \ldots + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A\cos \beta x + B \sin \beta x)$

Note: r is 0, 1, 2 … in such a way that there is no terms in $y_p(x)$ has the similar term as in the $y_c(x)$.

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Method of Variation of Parameters

The particular solution for $y''+by'+cy=g(x)$ (b and c constants) is given by $y(x)=u_1y_1+u_2y_2$, where;

$u_1=-\int\frac{y_2g(x)}{W}dx$ and $u_2=\int\frac{y_1g(x)}{W}dx$ $\quad W(y_1,y_2)=\begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}=y_1y'_2-y_2y'_1$

Laplace Transform

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$$

$f(t)$	$F(s)$
a	$\frac{a}{s}$
$t^n, n=1,2,3,...$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^nf(t), n=1,2,3,...$	$(-1)^n\frac{d^nF(s)}{ds^n}$
$H(t-a)$	$\frac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$y'(t)$	$sY(s)-y(0)$
$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

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Fourier Series

Fourier series expansion of periodic function with period 2 π

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

Half Range Series

$$a_0 = \frac{2}{L}\int_{0}^{L} f(x)dx$$

$$a_n = \frac{2}{L}\int_{0}^{L} f(x)\cos\frac{n\pi x}{L} dx$$

$$b_n = \frac{2}{L}\int_{0}^{L} f(x)\sin\frac{n\pi x}{L} dx$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$