Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS III
COURSE CODE	:	BDA 24003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **FIVE (5)** QUESTIONS ONLY

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

Q1 (a) Given function $f(x,y)=\sqrt{32-4x^2-8y^2}$

(i) Sketch the contour map where $k=0,\ \sqrt{7},\ \sqrt{16}$

(ii) Examine the domain and range for the function, f (x, y)

(iii) Sketch the surface, f (x,y)

(10 marks)

(b) Given $w(x,y,z)=2xyz$, $x=s^2+t^2$, $y=\frac{s}{t}$ and $z=\ln t$ Examine $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$ in terms of s and t.

(6 marks)

(c) Examine whether the limit $\lim_{(x,y)\to(0,0)}\frac{x^3y}{2x^6+y^2}$ exist or not by choosing two paths as follows. Justify your answer.

(i) along any linear line $y=mx$

(ii) along the curve $y=x^3$

(4 marks)

Q2 (a) Use the differential dz to approximate the change in $z=\sqrt{8-x^2-y^2}$ as (x,y) move from the point (1,1) to the point (0.99, 1.02). Compare this approximation change with the exact change in z.

(5 marks)

(b) Given that $w=r^2-r\tan\theta$ where $r=\sqrt{s}$ and $\theta=\pi s$. Solve $\frac{\partial w}{\partial s}$ in terms of s by using chain rule

(5 marks)

(c) At critical point for $f(x,y)=x^3-xy+y^3$. Examine whether each point is local maximum point, local minimum point or saddle point.

(10 marks)

Q3 (a) Solve the volume of the tetrahedron $3x+2y+4z=12$ in the first octant by using double integrals.

(5 marks)

(b) Examine $\iint x+y+2\,R\,dxdy$, where $\boldsymbol{R}$ is the region inside the unit square in which $x+y\geq 0.5$.

(5 marks)

(c) Solve the integral $\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-y^2-x^2}} z^2\sqrt{x^2+y^2+z^2}\,dzdydx$ to spherical coordinates by using the triple integral.

(10 marks)

Q4 (a) Given $\int_0^2\int_0^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{9-y^2-x^2}} \sqrt{x^2+y^2+z^2}\,dzdydx$

i. Sketch the solid represented by the triple integral and its projection on *xy*-plane.

(4 marks)

ii. Solve the integral to spherical coordinates. Then calculate the triple integral and show the answer in form of surd.

(6 marks)

(b) Given the triple integral, $\iiint_R \sqrt{x^2+y^2}\quad dV$

where R is the region lying above the *xy*-plane, and below cone $z=3-\sqrt{x^2-y^2}$

(i) Sketch the 3D-graph of the integral

(4 marks)

(ii) Exaamine the integral

(6 marks)

Q5 (a) Use polar coordinates to solve the shaded area in **Figure Q5 (a)** which is bounded outside of the graph of $r=2$ and inside of the graph of $r=4\sin\theta$.

(5 marks)

(b) Solve the surface area for the portion of the cone $z=\sqrt{x^2+y^2}$ that lies inside the cylinder $x^2+y^2=2x$.

(7 marks)

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(c) Examine the center of gravity of the triangular lamina with vertices (0, 0), (0, 1) and (1, 0) and the density function is $\delta(x, y) = xy$

(8 marks)

Q6 (a) The pressure, P, temperature, T, and volume, V, of an ideal gas are related by $PV = kT$, where $k > 0$ is a constant.

(i) By implicit differentiation, solve $\frac{\partial P}{\partial V}$, $\frac{\partial T}{\partial P}$ and $\frac{\partial V}{\partial T}$.

(ii) Solve that $\frac{\partial P}{\partial V}\frac{\partial T}{\partial P}\frac{\partial V}{\partial T} = -1$

(6 marks)

(b) Given a vector field $\boldsymbol{F}(x, y, z) = xy^2\,\mathbf{i} + y^3z\,\mathbf{j} + xz^3\,\mathbf{k}$, solve

(i) the divergence of F.

(ii) the curl of F.

(4 marks)

(c) Appraise Stoke's Theorem to examine $\iint_S (\nabla \times F).ndS$ where $\boldsymbol{F}(x, y, z) = (-y+z)\,\mathbf{i} + (x-z)\,\mathbf{j} + (x-y)\,\mathbf{k}$. S is the surface of upper hemisphere $z = \sqrt{1 - x^2 - y^2}$, oriented upward and C is the trace of S in the xy-plane.

(10 marks)

-END OF QUESTIONS –

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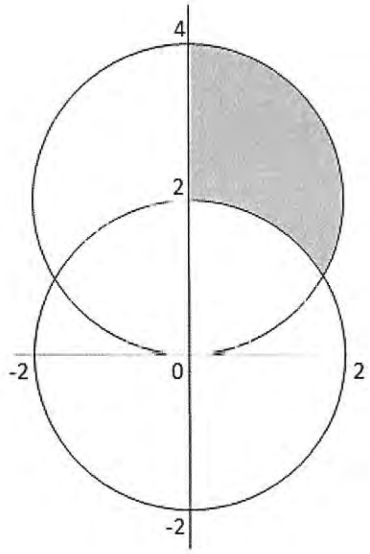

Figure Q5 (a)

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FORMULAE

Total Differential

For function $z = f(x, y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x, y)$, the relative change in z is given by

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$, where $F(x, y, f(x, y)) = 0$ for all (x, y) in the domain of f, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x, y)$ has a local maximum value at (a,b)
b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x, y)$ has a local minimum value at (a,b)
c. If $D < 0$
   $f(x, y)$ has a saddle point at (a,b)
d. If $D = 0$
   The test is inconclusive

Surface Area

$$\textit{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\,dA$$

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Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)rdzdrd\theta$$

Spherical Coordinates:

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2\sin\phi d\rho d\phi d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)dA$

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Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m},\frac{M_x}{m}\right)$$

Centroid

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\ of\ \ R}\iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\ of\ \ R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2\delta(x,y)dA$

b. $I_x = \iint_R y^2\delta(x,y)dA$

c. $I_o = \iint_R (x^2+y^2)\delta(x,y)dA$

In 3-D: Solid

Given that $\delta(x,y,z)$ is a density of solid

Mass, $m = \iiint_G \delta(x,y,z)dV$

If $\delta(x,y,z)=c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About *yz*-plane, $M_{yz} = \iiint_G x\delta(x,y,z)dV$

b. About *xz*-plane, $M_{xz} = \iiint_G y\delta(x,y,z)dV$

c. About *xy*-plane, $M_{xy} = \iiint_G z\delta(x,y,z)dV$

Centre of Gravity

$$(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{m},\frac{M_{xz}}{m},\frac{M_{xy}}{m}\right)$$

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Moment Inertia

a About x axis, $I_x = \iiint_G (y^2 + z^2)\delta(x,y,z)\,dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x,y,z)\,dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x,y,z)\,dV$

Directional Derivative

$D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j})\cdot\mathbf{u}$

Del Operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Gradient of $\phi = \nabla\phi$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla.\mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

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Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

Green's Theorem

$$\oint_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length,**

$$s = \int_a^b \|\mathbf{r}'(t)\| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt$$