CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME	:	BDD
EXAMINATION DATE	:	JULY 2020
DURATION	:	3 HOURS
INSTRUCTIONS	:	(a) ANSWER **ALL** QUESTIONS IN **PART A** (b) ANSWER **TWO (2)** QUESTIONS IN **PART B** PERFORM ALL CALCULATIONS IN FOUR (4) DECIMAL PLACES **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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PART A: ANSWER ALL QUESTIONS

Q1 (a) By using characteristic equation, find all the eigenvalues for the matrix A below

$$A = \begin{pmatrix} 0 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$

(5 marks)

(b) Given the matrix

$$A = \begin{pmatrix} a & -2 & 2 \\ -2 & 2 & -2 \\ 2 & -2 & b \end{pmatrix}$$

where
a - The last digit of your matrix number. If a =2, change the value to a=3.
b – The fifth digit of your matrix number. If b=2, change the value to b=3.

For example, a student with the matrix number CD150079 will have the values of $a = 9$ and $b = 7$.

Use the inverse power method to find the smallest eigenvalue, λ and its corresponding eigenvector, v of the matrix A using $v^{(0)} = (0 \quad 1 \quad 0)^T$, Stop the iteration until $|m_{k+1} - m_k| \leq 0.0005$.

(15 marks)

Q2 Solve the boundary-value problem

$$e^x y'' + xy' - 7(c + x)y = x^3$$

with the following boundary conditions

$$y(0) = 2, y(2) = 5$$

at $x = 0(0.4)2$ by using finite-difference method

where c is the last two digits of your matrix number.
For example, a student with the matrix number CD150079 will have the values of $c = 79$.

(20 marks)

Q3 The temperature distribution of a new heated plate of length 1 m and width of 2 m is under inspection. The boundaries of the plate are fixed at different temperatures and thus, heat flow from the hot to cold boundaries occurs. Within sufficient time elapses, the system will reach steady-state distribution of temperature. This case is governed by Laplace's equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0\leq x\leq 1, \quad 0\leq y\leq 2$$

with boundary conditions $u(0,y)=1, \quad u(1,y)=\sin(2\pi y), \quad 0\leq y\leq 2$

$$u(x,0)=u(x,2)=x(1-x), \quad 0\leq x\leq 1$$

Given that $\Delta x=0.5$ and $\Delta y=0.25$,

(a) Draw the finite difference grid into four boundaries to predict the temperature distribution. Label all unknown temperatures on the grid.

(7 marks)

(b) Solve the Laplace's equation by using the finite-difference method.

(13 marks)

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PART B: ANSWER TWO (2) QUESTIONS

Q4 (a) Given the graph of $f(x) = x - 2x\ln 2x$ as in **Figure Q4**. Find the root of $f(x)$ by using Bisection method with $|b_0 - a_0| = 1$ and iterate until $|f(c_i)| < \varepsilon = 0.0005$

(10 marks)

(b) Solve the system of linear equations below by Gauss Seidel iteration method.

$$x_1 + 2x_2 + 9x_3 = e$$
$$4x_1 - x_2 + 2x_3 = f$$
$$3x_1 + 6x_2 + x_3 = d$$

where

d - The first digit of your matrix number

e – The second digit of your matrix number

f – The fifth digit of your matrix number

For example, a student with the matrix number CD150079 will have the values of $d = 1, e = 5$ and $f = 7$.

Iterate until $\max_{1 \le i \le n} \left\{ \left| x_i^{(k+1)} - x_i^{(k)} \right| \right\} < 0.0005$

(10 marks)

Q5 (a) Given the set of data as shown in Table Q5(a).

Table Q5(a)

x	0	3	6	9
$y = f(x)$	2	-2	-2	2

Using cubic Lagrange interpolations, evaluate $f(4.5), f(7.5)$ and $f(9.75)$ if applicable.

(10 marks)

(b) Given the velocity of a motorcycle at a time t as in Table Q5(b):

Table Q5(b)

Time, $t(s)$	1.3	1.4	1.8	2.0
Velocity, $v(ms^{-1})$	11.2	12.0	14.2	15.7

Find the acceleration, $a = \frac{dv}{dt}$, of the motorcycle at $t = 1.6s$ using appropriate difference formulas.

(10 marks)

Q6 (a) Find the appropriate value for

(i) $$\frac{1}{\sqrt{2\pi}}\int_0^1 e^{\left(-x^2/2\right)}\,dx$$

(ii) $$\frac{1}{\pi}\int_0^{\pi}\cos\left(0.6g\sin x\right)\,dx$$

by using 3 point Gauss quadrature.

where
g - The last two digit of your matrix number

For example, a student with the matrix number CD150079 will have the values of $0.6g = 0.679$.

(10 marks)

(b) Given that $f(x) = e^x$. By taking $h = 10^{-k}$, where $k = 1, 2,$ find approximate values of $f''(2.2)$ using

(i) 3 point central difference formula,
(ii) 5 point difference formula.

Perform all the calculations in 6 decimal places.

(10 marks)

-END OF QUESTIONS-

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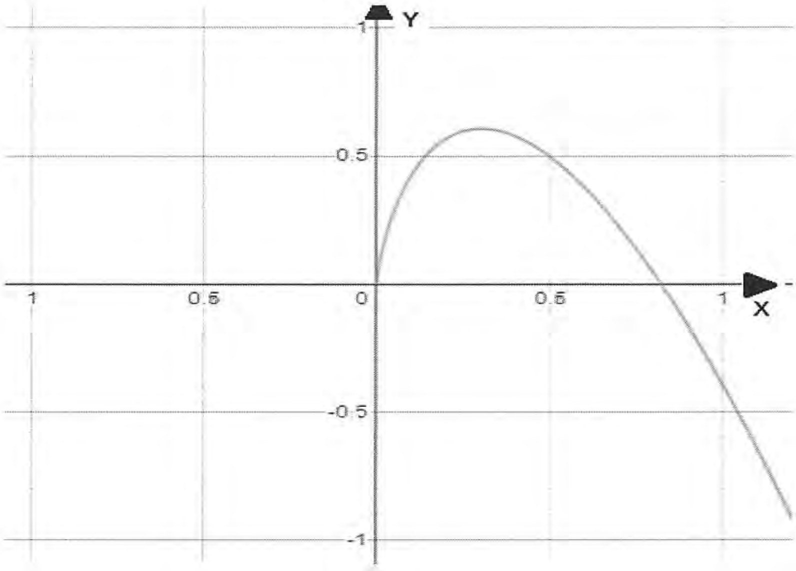

Figure Q4

FORMULA

Secant Method: $x_{i+1} = \dfrac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$

Thomas Method (Variant 1):

$$\begin{bmatrix} d_1 & e_1 & 0 & \cdots & 0 \\ c_2 & d_2 & e_2 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & d_{n-1} & e_{n-1} \\ 0 & 0 & \ldots & c_n & d_n \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 & 0 & \cdots & 0 \\ c_2 & \alpha_2 & 0 & \cdots & 0 \\ 0 & c_3 & \ddots & & 0 \\ \vdots & \vdots & & \alpha_{n-1} & 0 \\ 0 & 0 & \ldots & c_n & \alpha_n \end{bmatrix} \begin{bmatrix} 1 & \beta_1 & 0 & \cdots & 0 \\ 0 & 1 & \beta_2 & \cdots & 0 \\ 0 & 0 & \ddots & & 0 \\ \vdots & \vdots & & 1 & \beta_{n-1} \\ 0 & 0 & \ldots & 0 & 1 \end{bmatrix}$$

Lagrange interpolation:

$$f(x) = P_n(x) = \sum_{i=0}^{n} L_i(x) f_i \quad where \quad L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

Newton's Divided Difference interpolating polynomial:

$$f(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$

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FORMULA

Thomas Algorithm:

i	1	2	$\cdots$	n
d_i				
e_i c_i				
b_i				
$\alpha_1 = d_i$ $\alpha_i - d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

Inverse Power Method:

$$\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$$

Characteristic Equation:

$$\det(A - \lambda I) = 0$$

Euler 's Method:

$$f(x_i, y_i) = y'(x_i)\ ;\ y(x_{i+1}) = y(x_i) + hy'(x_i)$$

Modified Euler's Method:

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + \frac{1}{2}k_1 + \frac{1}{2}k_2$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + h, y_i + k_1)$$

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FORMULA

Gauss Quadrature:

$$x_\xi = \frac{1}{2}\left[(1-\xi)x_0 + (1+\xi)x_n\right]$$

$$I = \left(\frac{x_n - x_0}{2}\right)I_\xi$$

$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + K + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692	0.555555556
	0.0	0.888888889

First Derivative, $f'(x)$

2 point forward difference:
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

2 point backward difference:
$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

3 point central difference:
$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

3 point forward difference:
$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

3 point backward difference:
$$f'(x) \approx \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h}$$

5 point central difference:
$$f'(x) \approx \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$$

Second Derivative, $f''(x)$

3 point central difference:
$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

5 point central difference:
$$f''(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) \ldots - f(x+2h)}{12h^2}$$