UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER II
SESSION 2019/2020

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BDU 11003
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	JULY 2020
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} 0, & -\pi \leq x < -0.5\pi \\ 1, & -0.5\pi < x < 0.5\pi \\ 0, & 0.5\pi < x \leq \pi \end{cases}$$

and $f(x) = f(x + 2\pi)$.

(a) Sketch the graph of $f(x)$ over $-3\pi < x < 3\pi$.

(2 marks)

(b) Find the Fourier coefficients corresponding to $f(x)$.

(15 marks)

(c) From (b), prove that the Fourier series for $f(x)$

$$f(x) = \frac{1}{2} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cos(2n-1)\,x}{2n-1}.$$

(5 marks)

Q2 (a) Solve

$$(3x^2 - 2xy + e^y - ye^{-x})\,dx + (2y - x^2 + e^{-x} + xe^y)\,dy = 0$$

with initial value $y(0) = 1$.

(11 marks)

(b) According to Newton's law of cooling, the rate at which a body cools is given by the equation

$$\frac{dT}{dt} = -k(T - T_s),$$

where T_s is the temperature of the surrounding medium, k is a constant and t is the time in minutes. On 19th of February 2020 just before midday the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 21°C. At 12 noon the temperature of the body is 27°C and at 1 p.m. it is 24°C. Assume that the temperature of the body at the time of death was 37°C and that it has cooled in accord with Newton's Law. What was the time of death?

(9 marks)

PART B

Q3 (a) By using an appropriate method, solve

$$y'' - 2y' - 3y = 4e^{3x} + 9x$$

with $y(0) = 2$ and $y'(0) = -2$.

(13 marks)

(b) A mass of 20.4 kg is suspended from a spring with a known spring constant of 29.4 N/m. The mass is set in motion from its equilibrium position with an upward velocity of 3.6m/s. The motion can be described in the differential equation

$$\ddot{x} + \frac{k}{m}x = 0$$

where m is the mass of the object and k is the spring constant.

(i) Determine the initial conditions.

(1 mark)

(ii) Find an equation for the position of the mass at any time t.

(6 marks)

Q4 (a) Determine the Laplace transform for each of the following function.

(i) $(2 + t^3)e^{-2t}$.

(4 marks)

(ii) $\sin(t - 2\pi)\, H(t - 2\pi)$.

(4 marks)

(iii) $\sin 3t \;\; \delta(t - \pi)$.

(2 marks)

(b) Consider the periodic function

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1 - t, & 1 \le t < 2 \end{cases},$$
$$f(t) = f(t+2).$$

Sketch the graph of $f(t)$ and find its Laplace transform.

$$\left[\text{Hint: } \mathcal{L}[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t)\, dt, \;\; s > 0. \right]$$

(10 marks)

Q5 (a) (i) Find the inverse Laplace transform of

$$\frac{s+3}{s^2-6s+13}$$

(5 marks)

(ii) From **Q5(a)(i)**, find

$$\mathcal{L}^{-1}\left\{\frac{(s+3)e^{-\frac{1}{2}\pi s}}{s^2-6s+13}\right\}$$

(3 marks)

(b) (i) Express

$$\frac{1}{(s-1)(s-2)^2}$$

in partial fractions and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)^2}\right\} = e^t - e^{2t} + te^{2t}.$$

(7 marks)

(ii) Use the result in **Q5(b)(i)** to solve the differential equation

$$y' - y = te^{2t}$$

which satisfies the initial condition of $y(0) = 1$.

(5 marks)

Q6 (a) (i) Show that $\frac{dy}{dx} = \frac{y}{y-x}$ is a homogeneous differential equation.

(2 marks)

(ii) Hence, solve the differential equation in part **Q6(a)(i)**.

(10 marks)

(b) Analyze the given differential equation. Determine an appropriate method to solve it in order to obtain its particular solution

$$\frac{dy}{dx} = y\tan x - \sec x,\ y(0) = 1.$$

(8 marks)

-END OF QUESTIONS-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\}=\int_0^\infty f(t)e^{-st}\,dt=F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\}=\dfrac{1}{1-e^{-sT}}\displaystyle\int_0^T e^{-st}f(t)\,dt,\quad s>0.$

Fourier Series

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L}+b_n\sin\frac{n\pi x}{L}\right]$$

where

$$a_0=\frac{1}{L}\int_{-L}^{L}f(x)\,dx$$

$$a_n=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$$