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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	CALCULUS
COURSE CODE	:	BFC15003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY 2021 / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS

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THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1 (a) An object, initially at rest, falls due to gravity. Find its instantaneous velocity at $t = 3.8$ seconds and at $t = 5.4$ seconds. The function is given as $f(t) = 16t^2$.

(5 marks)

(b) Using the Product Rule, find all points of $y = \sin^2 x$ where the tangent line is horizontal.

(5 marks)

(c) Differentiate the following functions,

(i) $y = x^2 \cos x$

(2 marks)

(ii) $y = \sqrt{x^{2+\pi}}$

(4 marks)

(iii) $x^4 + \sin y = x^3 y^2$

(4 marks)

Q2 (a) Water is flowing out at the rate of 6 m^3/min from a reservoir shaped like hemispherical bowl of radius 13 m, shown in **Figure Q2(a)**. Given that the volume of water in a hemispherical bowl of radius R is $V = \left(\frac{\pi}{3}\right) y^2(3R - y)$ when water is y meters deep. Determine:

(i) rate of the water level changing when water is 8 m deep.

(5 marks)

(ii) the radius r of the water's surface when the water is y m deep.

(2 marks)

(iii) rate of the radius r changing when the water is 8 m deep.

(3 marks)

(b) A man in a rowboat at P as shown in **Figure Q2(b)** is 5 km away from the nearest point A on a straight shore, wishes to reach a point B, which is 6 km from point A along the shore ($AB = 6\ km$), in the shortest time. Calculate the location he should landed if he can row 2 km/h and walk 4 km/h.

(10 marks)

Q3 (a) Evaluate the following integral:

(i) $\int_1^5 (x^2+9)^2 \, dx$

(2 marks)

(ii) $\int_2^8 \frac{1}{\sqrt{x}} \, dx$

(2 marks)

(b) Evaluate $\int (6w+12) \sin^2\left(\frac{w}{9}\right) dw$ by using integration by parts technique.

(5 marks)

(c) Determine $\int \frac{12}{5-2x} \, dx$ by using substitution technique.

(5 marks)

(d) Integrate the following using integration by partial fractions method;

(i) $\int \frac{2x^2-7x-21}{(x+2)(x-1)(x+4)} \, dx$

(2 marks)

(ii) $\int \frac{6x+11}{(x-10)^2} \, dx$

(4 marks)

Q4 (a) Find the area bounded below by $f(x) = -x^2+4x+3$ and above by $g(x) = -x^3 + 7x^2 - 10x + 5$ over the interval $1 \le x \le 2$. Sketch the graph to show the region.

(10 marks)

(b) Determine the surface area of the solid generated by rotating $y = \sqrt{9-x^2}, -2 \le x \le 2$ about the x-axis.

(10 marks)

Q5 (a) Show that every member of the family of functions $y = \frac{c}{x} + 2$ is a solution of the first order differential equation $\frac{dy}{dx} = \frac{1}{x}(2-y)$ on the interval $(0, \infty)$, where C is any constant.

(4 marks)

(b) Separable is type of the First Order Differential Equations, determine whether the following equations are separable;

(i) $\frac{dy}{dx} - xy = x$

(2 marks)

(ii) $\frac{dy}{dx} = xe^{y-x}$

(2 marks)

(iii) $\sin y \cos x \frac{dy}{dx} - \cos y \sin x = 0$

(2 marks)

(iv) $x\frac{dy}{dx} = x - 2y$

(2 marks)

(c) Homogeneous is a type of the First Order Differential Equations, Determine whether the following equations are homogeneous;

(i) $\frac{dy}{dx} = \frac{y-x}{y+x}$

(2 marks)

(ii) $\frac{dy}{dx} = x - y$

(2 marks)

(iii) $\frac{dy}{dx} = \frac{xy}{x^2-y^2}$

(2 marks)

(iv) $y\frac{dy}{dx} = x(lny - lnx)$

(2 marks)

– END OF QUESTIONS –

FINAL EXAMINATION

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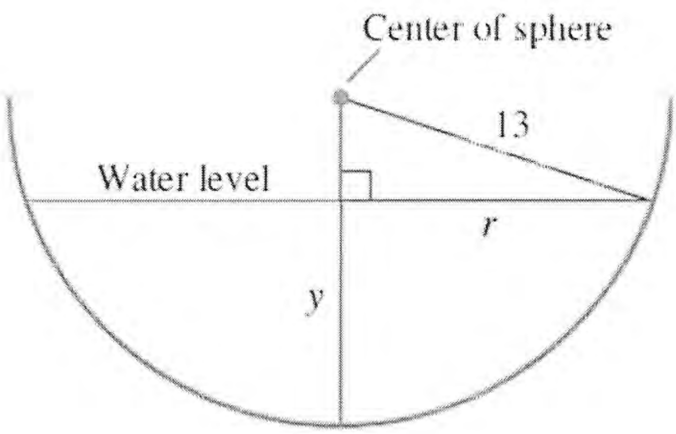

FIGURE Q2(a)

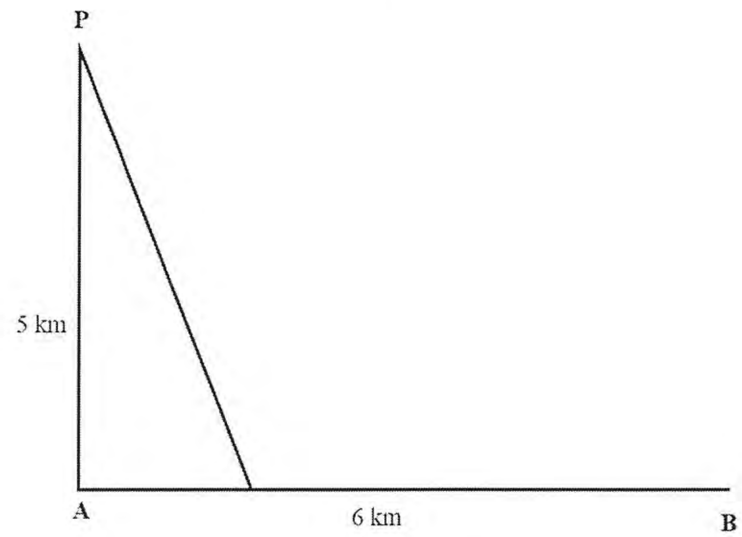

FIGURE Q2(b)

FINAL EXAMINATION

SEMESTER/ SESSION: SEM II 2020/2021 PROGRAMME: BFF
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Formulae

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$, k constant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C$, $n \neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec } x]=-\text{cosec } x \cot x$	$\int \text{cosec } x \cot x\,dx = -\text{cosec } x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech } x]=-\text{sech } x \tanh x$	$\int \text{sech } x \tanh x\,dx = -\text{sech } x + C$
$\frac{d}{dx}[\text{cosech } x]=-\text{cosech } x \coth x$	$\int \text{cosech } x \coth x\,dx = -\text{cosech } x + C$

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Formulae

Trigonometric	Hiperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \operatorname{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	
Logarithm	**Inverse Hiperbolic**
$a^x = e^{x \ln a}$	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x.
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$
	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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Formulae

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\kappa = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\kappa}$

Curvature of parametric curve

Curvature, $\kappa = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\kappa}$