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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWA 20303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 (a) According to Newton's law of cooling, the rate at which a body cools is given by the equation

$$\frac{dT}{dt} = -k(T - T_s),$$

where T_s is the temperature of the surrounding medium, k is a constant and t is the time in minutes. If the body cools from 80°C to 50°C in 20 minutes with the surrounding temperature of 10°C, how long does it take for the body to cool from 80°C to 30°C?

(8 marks)

(b) By using method of variation of parameters, find the solution of the differential equation

$$y'' - 2y' - 3y = \frac{64x}{e^{-x}}.$$

(12 marks)

Q2 (a) State the difference between the ordinary differential equations and partial differential equations in terms of the independent variable. Give example for each equation.

(2 marks)

(b) Identify the order, the degree and the independent variable of the following differential equation

$$\left(\frac{d^4 r}{dh^4}\right)^3 + \left(\frac{dr}{dh}\right)^5 + r^3 = e.$$

(3 marks)

(c) The general equation describing the mass-spring system is

$$m\frac{d^2 x}{dt^2} + b\frac{dx}{dt} + kx = F(t).$$

A spring is stretched 0.2 m $(= \Delta l)$ when 6 kg of iron ball is attached. The weight is then pulled down an additional 0.3 m and released with an upward velocity of 4.5 m/s. Determine an equation for the position of the spring when the free vibration has a damping constant of 40.

(10 marks)

Q3 (a) Show that

$$\int_0^\infty (3t^2+t+2)\delta(3t-1)dt = \frac{1}{9}\int_0^\infty (u^2+u+6)\delta(u-1)du.$$

Hence, compute the integrals.

(5 marks)

(b) Find $\mathcal{L}\{e^{-t}(\sin(2t)+\cos(2t))^2\}$.

(7 marks)

(c) A damped force oscillation is given by

$$y''+4y'+4y=f(t), \quad y(0)=0 \text{ and } y'(0)=0,$$

where

$$f(t)=\begin{cases}0, & 0 \le t < 2,\\ e^{-(t-2)}, & t>2.\end{cases}$$

By using Laplace Transform, solve for $y(t)$.

(13 marks)

Q4 (a) Show that the solutions of the first order differential equation $y'=2xy$ by using both

(i) separating variable method and

(ii) power series method,

are the same.

(10 marks)

(b) By using an appropriate power series method, determine the solution to the given equation up to x^3 only.

$$y'+e^{-x}y=x^3, \quad y(0)=3.$$

(10 marks)

Q5 (a) Solve the given system of first order differential equations

$$y_1' = 4y_1 + 2y_2,$$
$$y_2' = 3y_1 + 3y_2.$$

(8 marks)

(b) By using the Laplace transform, find the following system of linear differential equations

$$x' + x - y = 0,$$
$$y' - x + y = 2,$$

subject to initial conditions $x(0) = 1, \quad y(0) = 2.$

(12 marks)

- END OF QUESTIONS -