CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION (ONLINE) SEMESTER I SESSION 2020/2021

COURSE NAME	:	TECHNIQUES OF OPTIMIZATION II
COURSE CODE	:	BWA 40703
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

CONFIDENTIAL

Q1 (a) Define a penalty objective function and the related penalty function for the problem

$$\begin{aligned} &\text{Minimize} && f(x) \\ &\text{subject to} && g_i(x) \le 0, i = 1, 2, \cdots, p. \end{aligned}$$

(6 marks)

(b) Express a formula to the decision variables for the penalty objective function

$$\text{Minimize} \quad q(c, x_1, x_2) = x_1^2 + x_2^2 + \frac{c}{2}(1 - x_1 - x_2^2)^2$$

where c is a positive constant, and deduce an optimal solution as c approaches to a large number.

(14 marks)

(c) Identify the Hessian for the penalty objective function in **Q1(b)**.

(5 marks)

Q2 (a) Describe the primal function and the dual function, then, provide the results given by the Weak Duality Proposition and the Strong Duality Theorem for the problem

$$\begin{aligned} &\text{Minimize} && f(x) \\ &\text{subject to} && g(x) \le 0,\ h(x) = 0 \ \ x \in \Omega. \end{aligned}$$

(6 marks)

(b) Determine the gradient and Hessian of the dual function.

$$\phi(\lambda) = f(x) + \lambda^T h(x).$$

(11 marks)

(c) Justify **TWO (2)** types of the problem from the augmented Lagrangian for the equality constrained problem. Next, summarize a typical calculation procedure step and outline the updating rule in the augmented Lagrangian method.

(8 marks)

Q3 (a) Define a merit function from a standard nonlinear programming problem

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & g(x) \leq 0,\ h(x) = 0, \\ & x \in \Omega \subset \Re^n,\ m \leq n. \end{array}$$

(4 marks)

(b) Discuss a global minimum point of the merit function.

(8 marks)

(c) Revise the Newton equations for the merit function.

(13 marks)

Q4 (a) Explain the calculation procedure for using the gradient projection method to the problem

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{subject to} & Ax \leq b \text{ and } Qx = q. \end{array}$$

(11 marks)

(b) Validate the point before running the next iteration for the following optimization problem using the gradient projection method, where the current point is $(2.3626, 0.9091, 1.3636, 0.0000)^{\mathrm{T}}$.

$$\begin{array}{ll} \text{Minimize} & x_1^2 + x_2^2 + x_3^2 + x_4^2 - 2x_1 - 3x_4 \\ \text{subject to} & 2x_1 + x_2 + x_3 + 4x_4 = 7 \\ & x_1 + x_2 + 2x_3 + x_4 = 6 \\ & x_i \geq 0,\ i = 1,2,3,4. \end{array}$$

(14 marks)

– END OF QUESTIONS –