UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(ONLINE)
SEMESTER I
SESSION 2020/2021

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	DAU 34403
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JANUARY / FEBRUARY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS IN PART B

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THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A

Q1 (a) (i) Based on $L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$. Find $F(s)$ for:

$$f(t) = t - 4$$

(5 marks)

(ii) Determine the Laplace transform of $f(t) = \begin{cases} 0 & if & 0 \le t \le 1 \\ 1 & if & \le t \le 2 \\ 2 & if & 2 \le t \end{cases}$.

(8 marks)

(b) (i) By using Linearity Theorem, find the Laplace transform of:

$$L\{3e^{4t} - 6 + 5\cos 2t\}.$$

(2 marks)

(ii) By using First Shifting Theorem, find the Laplace transform of:

$$L\{t^2 e^{3t}\}.$$

(2 marks)

(iii) By using Multiply with t^n, find the Laplace transform of:

$$L\{te^{3t}\sin 2t\}.$$

(3 marks)

Q2 (a) Find the inverse of the following Laplace expression:

(i) $\dfrac{6s+3}{s^2+25}$.

(4 marks)

(ii) $\dfrac{1}{s^4} + \dfrac{1}{2s+8} - \dfrac{4}{s-3}$.

(4 marks)

(ii) $\dfrac{8}{3s^2+12} - \dfrac{3}{s^2-49}$.

(5 marks)

(b) (i) Express $\dfrac{s+4}{s^2-2s-3}$ as partial fractions.

(5 marks)

(ii) Find the inverse Laplace of the partial fraction from **Q2(b)(i).**

(2 marks)

PART B

Q3 (a) By using Laplace Transform, find the solution $y(t)$ for each of the following differential equation by using.

(i) $y' + y = e^{-4t}, \quad y(0) = 2.$

(8 marks)

(ii) $y'' - 6y' + 8y = 0 \qquad y(0) = 0 \qquad y'(0) = 3$

(12 marks)

Q4 (a) Given $\left(3x^2 + \frac{7}{3}xy^3\right)dx + \left(\frac{7}{2}x^2y^2 - 2y^2\right)dy = 0.$

(i) Show that the differential equation above is an exact equation.

(3 marks)

(ii) Then, solve the equation from **Q4 (a)(i).**

(8 marks)

(b) Given a first order differential equation:

$$(xy)dy - (y^2 - x^2)dx = 0.$$

(i) Show that the differential equation above is a homogenous equation.

(2 marks)

(ii) Solve the homogenous equation with condition $y(1) = 0$.

(7 marks)

Q5 (a) In a particular culture, the rate of increase of bacteria is proportional to the number of bacteria P, present at time t hours after the experiment. Given that the number of bacteria at the beginning is 10^6, and after 1 hour is 10^9, find:

(i) an expression for the number of bacteria.

(5 marks)

(ii) the number of bacteria after 5 hours

(2 marks)

(iii) time taken for the number of bacteria to be 3 times the original.

(3 marks)

(b) For coffee in a ceramic cup, suppose $k = 0.05$ with time measured in minutes

(i) By using Newton's Law of cooling, find the cooling equation of the body

(1 mark)

(ii) Use Newton's Law of Cooling to predict the temperature of the coffee, initially at a temperature of 200^{o}F, that is left to sit in a room at 75^{o}F for 15 minutes.

(4 marks)

(iii) Find the time taken for the coffee to reach 100^{o}F.

(5 marks)

Q6 (a) Given $y''-y'-2y=0$ with $y(0)=2$ and $y'(0)=1$.

(i) Find the general solution using second order homogenous differential equation.

(3 marks)

(ii) Hence, solve the initial value problem.

(7 marks)

(b) Given $y''-4y=e^{2x}$.

(i) Using method of undetermined coefficient, find the particular solution, y_p.

(3 marks)

(ii) Hence, obtain the general solution.

(7 marks)

Q7 Find the general solution of the following second order non-homogeneous differential equation by using the method of variation parameters.

(a) $$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 4e^{2x}$$

(9 marks)

(b) $$\frac{d^2y}{dx^2} + 4y = 3\cos 2x$$

(11 marks)

–END OF QUESTIONS–

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Table 1: Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$

Characteristic equation : $am^2 + bm + c = 0$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2: Method Of Undetermined Coefficients

Case	$F(x)$	$y_p(x)$
1	Simple polynomial: $A_0 + A_1x + \ldots + A_nx^n$	$x^r(B_0 + B_1x + \ldots + B_nx^n), r = 0,1,2,\ldots$
2	Exponential function: $Ce^{\alpha x}$	$x^r(Ke^{\alpha x}), r = 0,1,2,\ldots$
3	Simple trigonometry: $C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x), r = 0,1,2,\ldots$

Table 3: Method of Variation of Parameters

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx$ $u_2 = \int \frac{y_1 f(x)}{aW} dx$

Particular solution, $y_p = u_1y_1 + u_2y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$

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Table 4: Trigonometry Identities

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x\sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x\cos y = \cos(x+y) + \cos(x-y)$$

Table 5: Differentiation and Integration

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln \lvert x \rvert + C$
$\frac{d}{dx}\ln\lvert ax+b\rvert = \frac{1}{ax+b}$	$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln\lvert ax+b\rvert + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin ax = a\cos ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\frac{d}{dx}\cos ax = -a\sin ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$

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Table 6: Laplace and Inverse Laplace Transforms

$$L\{f(t)\} = \int_0^{\infty} f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$

The First Shift Theorem

$e^{at}f(t)$	$F(s-a)$

Multiply with t^n

$t^n f(t), n = 1, 2, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$

Initial Value Problem

$$L\{y(t)\} = Y(s)$$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

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Table 7: Formula for Growth and Decay and Newton's Cooling Law

Growth and Decay

$$N = Ae^{-kt}$$

Newton's Cooling Law

$$T = (T_0 - T_s)e^{-kt} + T_s$$