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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
(TAKE HOME ASSESSMENT)
SEMESTER II
SESSION 2020/2021

COURSE NAME	:	MULTIVARIABLE CALCULUS/ENGINEERING MATHEMATICS III
COURSE CODE	:	BEE 20303/BEE 21503
PROGRAMME CODE	:	BEV / BEJ
EXAMINATION DATE	:	JULY 2021
DURATION	:	3 HOURS
INSTRUCTION	:	ANSWER **ALL** QUESTIONS **OPEN BOOK EXAMINATION**

THIS QUESTION PAPER CONSISTS OF **FIVE(5)** PAGES

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Q1 (a) Compute the multiple integrals of

(i) $$\int_0^1 \int_0^{\sqrt{4-z^2}} \int_0^y xy \, dxdydz$$

(5 marks)

(ii) $\iint_R (8 - x^2 - y^2)dA$, where $R = \{(x, y) = -1 \leq x \leq 1 \; and \; 0 \leq y \leq 2\}$

(5 marks)

(b) Evaluate the following triple integral by using spherical coordinates.

$$I = \int_{-4}^{4} \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} (3x^2 + 3y^2 + 3z^2) \, dzdydx$$

(5 marks)

(c) (i) By using polar coordinate, evaluate $\iint_R \; x^2 + y^2 \, dA$, if R is the region in between circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first octant.

(5 marks)

(ii) By using double integrals, find the area of the regions enclosed by $y = \sin x, y = \cos x, x = \frac{\pi}{4}, x = \frac{\pi}{2}$.

(5 marks)

Q2 (a) Find the first partial derivative of

(i) $f(x, y, z) = \frac{x^2 y}{z^3}$

(3 marks)

(ii) $f(x, y, z) = e^{yz} \cos xy$

(3 marks)

(b) Solve $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ by applying chain rule if $z = \ln(x^2 - 2y)$, and $x = u + v$, $y = e^v$.

(6 marks)

(c) Show that the function $z = e^x \sin y + e^y \cos x$ satisfies Laplace equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

(5 marks)

(d) For the following equations, find $\frac{\partial z}{\partial x}$ if $z = f(x, y)$ is implicitly defined as a function of x and y.

(i) $ye^{xz} - 3ye^{yz} = -4ze^{xy} + 1$

(4 marks)

(ii) $2x^2z^4 - 5\ln(xyz^2) = 4z$

(4 marks)

Q3 (a) Calculate the divergence of curl of vector $\vec{\boldsymbol{F}} = e^{2z+y}\,\hat{\imath} + \sin yz^2\,\hat{\jmath} + \cos yzx\,\hat{\boldsymbol{k}}$, $\nabla \cdot (\nabla \times \vec{\boldsymbol{F}})$.

(5 marks)

(b) Find the work done by force field $\vec{\boldsymbol{F}} = 3x^2\,\hat{\imath} + y^2\hat{\jmath}$ on a particle when it moves from (0, 0) to (-π, 0) along the curves C1 and C2 in **Figure Q3 (b)** by solving $\int_r \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$. Based on your calculation, judge whether the force, $\vec{\boldsymbol{F}}$ is conservative or non-conservative and give your explanation.

(10 marks)

(c) Evaluate $\iint_\sigma 2y^2 + 4z^2\,ds$, where σ is the first octant portion of a cone $z = \sqrt{x^2 + y^2}$ with height of 9.

(10 marks)

Q4 (a) Define Gauss's theorem and Stokes' theorem.

(4 marks)

(b) Consider the outward flux of vector field,

$$\vec{\boldsymbol{F}}(x, y, z) = y\hat{\imath} + xy\hat{\jmath} + z\hat{\boldsymbol{k}}$$

across the surfaces enclosed by cylinder $x^2 + y^2 = 4$, z = 4 plane z = 0, and cone $z = \sqrt{x^2 + y^2}$.

(i) Sketch the graph of the surfaces.

(2 marks)

(ii) Find the divergence of $\vec{\boldsymbol{F}}(x, y, z)$.

(2 marks)

(iii) Use Gauss's theorem to evaluate the flux of the vector field $\vec{\boldsymbol{F}}(x, y, z)$ across the surfaces.

(7 marks)

(c) A vector field oriented outward is given by

$$\vec{\boldsymbol{F}}(x,y,z) = -\frac{3}{2}y^2\hat{\boldsymbol{\imath}} - 2xy\hat{\boldsymbol{\jmath}} + yz\hat{\boldsymbol{k}}$$

Suppose that the surface of the plane $x + y + z = 1$ in the first octant contained within triangle C with vertices (1, 0, 0), (0, 1, 0) and (0, 0, 1) transverse in anticlockwise direction.

(i) Sketch the surface of the plane.

(2 marks)

(ii) Evaluate $\oint_C \vec{\boldsymbol{F}} \cdot \overrightarrow{d\boldsymbol{r}}$ by C using Stoke's theorem.

(8 marks)

– END OF QUESTIONS –

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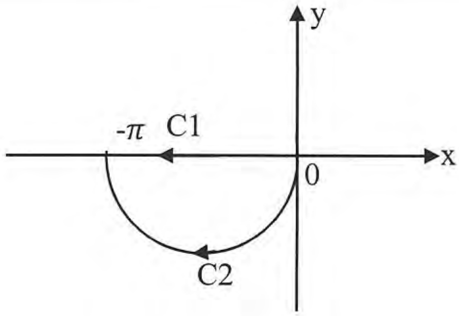

FIGURE Q3(b)