Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	NUMERICAL METHODS/ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE 32402/BEE 31602
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWERS **ALL** QUESTIONS.
		2. THIS FINAL EXAMINATION IS A **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) Find value I_d , that you will use it in **Question 1(b)** and **1(c)**

$$I_d = \frac{m}{d}$$

Note: m is your month of birth and d is your day of birth

(1 mark)

(b) The following data given are collected from the experimental measurement on the electrical circuit with the current as input variable and voltage as output variable:

i	0	1.2	2.4	3.6	4.8	6	7.2
$v(i)$	$2I_d$	$9I_d$	$18I_d$	$27I_d$	$38I_d$	$45I_d$	$57I_d$

Compare **FOUR(4)** suitable methods to approximate $v'(6)$ in 4 decimal places in which the exact solution of $v'(6)$ is 8.3878.

(15 marks)

(c) A point P is moving along the curve whose equation is $f(x) = I_d e^{2x+3}$. By using second derivative for 3-point central and 5-point difference formula with $h = 0.05$, calculate how far P is moving when $x = 1.25$.

(9 marks)

Q2 (a) A miniature quadcopter flies in a curve line from one point to another. Suppose that the path of the quadcopter from its initial hovering point to the final resting point is described by:

$$y = 2.15 + 2.09x - 0.41x^2, 0 \le x \le 3.6$$

where x is the horizontal distance (in meters) from the point of release, and y is the total distance (in meters) from the initial point.

(i) Estimate the travels distance of the quadcopter from the moment of its hovering point to the moment it rests, by using trapezoidal rule and appropriate Simpson's rule with $h = 0.4$ given that the arc length of the curve line is:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

(11 marks)

Estimate (C2), change to TOS

(ii) Calculate the exact solution of the traveled distance by using a scientific calculator.

(1 mark)

(iii) Find the absolute error for each methods (from **Q2(a)(i)**).

(2 mark)

(iv) Determine which method approximates better.

(1 mark)

(b) Evaluate the following equation by using Simpson's 3/8 rule with 12 equal parts of interval subdivision. The value of I_d is same as in equation **Q1(a)**. Find the value of x and its absolute errors.

$$\int_{0.2}^{1.4} I_d e^{\sin x}\, dx$$

(10 marks)

Q3 (a) A polluted pool has an initial concentration of bacteria of 10^4 part/m^3, while the acceptable level is only 5×10^3 part/m^3. The concentration of the bacteria will reduce as fresh water enter the pool. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by:

$$\frac{dC}{dt} + 0.05C = 0, \qquad C(0) = 10^4$$

(i) Estimate the concentration of the pollutant after 15 weeks by taking a step size of 1 week.

(11 marks)

(ii) Find the absolute error if the exact solution of **Q3(a)(i)** is

$$C(t) = 10^4 e^{-0.05t}$$

(2 marks)

(b) Find the solution for $\cos(x)\frac{d^2y}{dx^2} - (1+x^2)\frac{dy}{dx} - 2xy = \frac{1}{1+x^2}$ with the conditions $y(1) = -1,\ y(2) = 2$ and $h = 0.25$ by using finite-difference method.

(12 marks)

Q4 (a) The temperature distribution $u(x,t)$ of the one-dimensional gold rod is governed by the heat equation as follows.

$$\frac{\partial u}{\partial t} = 0.25\frac{\partial^2 u}{\partial x^2}$$

Given the boundary conditions $u(0,t) = 2t^2$, $u(1,t) = 5t$, for $0 \le t \le 0.04\ s$ and the initial condition $u(x,0) = x(1-x)$ for $0 \le x \le 1.0\ mm$, analyze the temperature distribution of the rod with $\Delta x = 0.25$ mm and $\Delta t = 0.02$ s in 4 decimal places.

(10 marks)

(b) An elastic string which is fixed at both ends is governed by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1,\ t \ge 0,$$

Where, $u(x,t)$ is the displacement of the string. The initial conditions are given by

$$u(x,0) = \begin{cases} x, & 0 \leq x \leq 0.5 \\ -(x-1) & 0.5 < x \leq 1 \end{cases} \qquad \frac{\partial u}{\partial t}(x,0) = 0, \quad 0 \leq x \leq 1$$

Determine the variation of the displacement of the string by using the finite-difference method for $0 \leq t \leq 0.3\ s$ using $\Delta x = 0.25\ mm$ and $\Delta t = 0.1\ s$.

(15 marks)

- END OF QUESTIONS -