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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	CIVIL ENGINEERING MATHEMATICS IV/ NUMERICAL METHOD
COURSE CODE	:	BFC 24203/ BFC 25203
PROGRAMME	:	BFF
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Choose an appropriate set (**SET A** or **SET B**) by giving reason and analyse the system of linear equations of the chosen set by using Gauss-Seidel iteration method. Use an initial guess of x=1, y=1, and z=1. Do all calculations in 3 decimal places.

SET A

$$\begin{bmatrix} -3 & 1 & 1 \\ 3 & 2 & -7 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

SET B

$$\begin{bmatrix} 3 & 2 & -1 \\ -1 & 3 & 2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ -1 \end{bmatrix}$$

(10 marks)

(b) Given a function of 1 meter length steel beam's defection as follow:

$$f(x) = e^{-x} + a,$$

where a is the last digit of your matrix number (example: AFxxxxx***a***).

(i) Interpret the above polynomial and complete the **Table Q1(b)**. All data must be in 4 decimal places.

(4 marks)

(ii) From the answer in **Table Q1(b)**, estimate the deflection of the beam at 0.4 meter by using Newton's divided-difference method.

(4 marks)

(iii) If a data $(0.5, e^{-x} + a)$ is added into the **Table Q1(b)**, estimate the new deflection value of the beam at 0.4 meter.

(7 marks)

Q2 (a) A cubic function is given, as follow:

$$f(x) = (x+3)^2(x-2)$$

(i) By taking $h = 0.1$, determine **SIX (6)** approximation values of the slope function at $x = -1.1$ using appropriate difference formula. Do all calculations in 3 decimal places.

(14 marks)

(ii) Based on answer in **Q2(a)(i)**, suggest the method that provide the best approximation.

(1 mark)

(b) Given the equation of the irregular curve of stream, $y = 16x^2 \sin(x)$. Approximate the stream cross-sectional area of irregular shapes from $x = 0$ to $x = \frac{\pi}{2}$ into 5 equal intervals by using accurate Simpson's rule and express the absolute error. Do all calculation in 3 decimal places.

(10 marks)

Q3 (a) The force on a yacht mast can be presented by the following function:

$$F = \int_0^H 200\left(\frac{z}{7+z}\right) e^{-2z/H}\, dz$$

where z is the elevation above the deck and H is the height of the mast. By considering $H = 30 + m$, which m is referring to the last digit of your matrix number (example: AFxxxxx***m***), compute F using 2-point and 3-point Gauss quadrature. Do all calculation in 4 decimal places.

(13 marks)

(b) Given matrix T:

$$T = \begin{pmatrix} 3 & 2 & 3 \\ 2 & 6 & 6 \\ 3 & 6 & t \end{pmatrix};\ \text{where } t = [(last\ number\ of\ your\ Matric\ No.) + 2]$$

(i) Find characteristic equation for matrix T

(3 marks)

(ii) Determine the dominant eigenvalue, and its corresponding eigenvectors using the Power Method. Let $\mathbf{v}^{(0)} = (1 \quad 1 \quad 1)^T$ and stop the iteration until $|m_{k+1} - m_k| \leq 0.0005$ or after five iterations, whichever comes first. Do all calculations in 4 decimal places.

(9 marks)

Q4 (a) The problem of noise pollution in Kuala Lumpur has become more serious due to the development of transportation systems, construction, and industrial activities. Malaysia's Department of Environment (DOE) had imposed a noise limit of 60 decibels (dB) to protect the population from this urban noise. The noise level in Kuala Lumpur is given by the following differential equation:

$$\frac{dL}{dt} = \frac{1}{0.001 \cdot a \cdot \ln(10) \cdot L}$$

where L is the sound pressure level in decibel (dB) and a is the last digit of your matric number (for example: AFxxxxx***a***). If the last digit of your number is zero (0) then take $a = 1$.

(i) Estimate the noise level in Kuala Lumpur from year 2022 to 2030 by using the fourth order Runge Kutta method with the increment of 2 years. Assume that the noise level in 2022 is 50.0 dB. Do all calculations in 3 decimal places.

(10 marks)

(ii) Based on the answer in **Q4 (a)**, are the noise levels acceptable in future built environment of Kuala Lumpur? Justify your answer.

(2 marks)

(b) Given the partial differential equation (PDE) that use for bearing capacity resistance of 10m pile foundation in 10 minutes is

$$\frac{\partial^2 u}{\partial t^2} - 6\frac{\partial^2 u}{\partial x^2} = 0$$

(i) Based on the criteria of variables and partial derivatives, justify the category of the above PDE.

(3 marks)

(ii) Using the above equation for $0 < x < 10,\ 0 < t < 10$, with boundary conditions, $u(0,t) = u(10,t) = 0$ for $0 \le t \le 10$, estimate the bearing capacity of the pile by using the explicit difference method. Assume that the initial conditions $u(x,0) = sin\pi x,\ \frac{\partial u(x,0)}{\partial t} = sin4\pi x$ for $0 \le x \le 10,\ h = \Delta x = 2.5$m and $k = \Delta t = 5$ minutes. Do all calculations in 3 decimal places.

(10 marks)

- END OF QUESTIONS -

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Table Q1(b): Polynomial for steel beam deflection.

x	**0**	**0.25**	**0.75**	**1.0**
$f(x) = e^{-x} + a$				

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FORMULAS

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij} x_j^{(k)}}{a_{ii}}$

Interpolations

Newton's divided difference : $f_i^{[0]} = f_i, \ \ i = 0, 1, \cdots, n$

$$f_i^{[j]} = \frac{f_{i+1}^{[j-1]} - f_i^{[j-1]}}{x_{i+j} - x_i}, \ \ j = 1, 2, \cdots, n$$

$$f(x) \approx P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \cdots f_0^{[n]}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Numerical Differentiation

First derivative,

2-point foward difference : $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2 – point backward difference : $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3 – point central difference : $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3 – point forward difference : $f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

3 – point backward difference : $f'(x) \approx \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h}$

5 – point central difference : $f'(x) \approx \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$

Second derivatives,

3 – point central difference : $f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$

5 – point central difference : $f''(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h)}{12h^2}$

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Numerical integration

Simpson's $\frac{1}{3}$ rule : $A_1 = \int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2)$

$$\int_{b=x_n}^{b=x_n} f(x)dx \approx \frac{h}{3}\left(f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i \right)$$

Simpson's $\frac{3}{8}$ rule : $A_1 = \int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3)$

$$\int_{b=x_n}^{b=x_n} f(x)dx \approx \frac{3h}{8}\left(f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{\left(\frac{n}{3}\right)-1} f_{3i} \right)$$

2-point Gauss Quadrature : $\int_{-1}^{1} g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature : $\int_{-1}^{1} g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigenvalues

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \ k = 0, 1, 2 \cdots$

Ordinary-differential Equations (ODEs)

Fourth-Order Runge-Kutta Method (RK4) : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right)$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equations (PDEs)

Second Order Linear PDEs : $A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu = G$

Wave Equation : $\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j}$

$$\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j-1}}{k^2} = c^2\left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$