



UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022**

- COURSE NAME : CIVIL ENGINEERING
MATHEMATICS IV/
NUMERICAL METHOD
- COURSE CODE : BFC 24203/ BFC 25203
- PROGRAMME : BFF
- EXAMINATION DATE : JULY 2022
- DURATION : 3 HOURS
- INSTRUCTION
1. ANSWER ALL QUESTIONS
 2. THIS FINAL EXAMINATION IS AN **ONLINE ASSESSMENT AND CONDUCTED VIA CLOSED BOOK.**
 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

- Q1** (a) Choose an appropriate set (**SET A** or **SET B**) by giving reason and analyse the system of linear equations of the chosen set by using Gauss-Seidel iteration method. Use an initial guess of $x=1$, $y=1$, and $z=1$. Do all calculations in 3 decimal places.

$$\text{SET A} \quad \begin{bmatrix} -3 & 1 & 1 \\ 3 & 2 & -7 \\ 2 & -5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\text{SET B} \quad \begin{bmatrix} 3 & 2 & -1 \\ -1 & 3 & 2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ -1 \end{bmatrix}$$

(10 marks)

- (b) Given a function of 1 meter length steel beam's deflection as follow:

$$f(x) = e^{-x} + a,$$

where a is the last digit of your matrix number (example: AFxxxxxa).

- (i) Interpret the above polynomial and complete the **Table Q1(b)**. All data must be in 4 decimal places.

(4 marks)

- (ii) From the answer in **Table Q1(b)**, estimate the deflection of the beam at 0.4 meter by using Newton's divided-difference method.

(4 marks)

- (iii) If a data $(0.5, e^{-x} + a)$ is added into the **Table Q1(b)**, estimate the new deflection value of the beam at 0.4 meter.

(7 marks)

- Q2** (a) A cubic function is given, as follow:

$$f(x) = (x + 3)^2(x - 2)$$

- (i) By taking $h = 0.1$, determine **SIX (6)** approximation values of the slope function at $x = -1.1$ using appropriate difference formula. Do all calculations in 3 decimal places.

(14 marks)

- (ii) Based on answer in **Q2(a)(i)**, suggest the method that provide the best approximation.

(1 mark)

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- (b) Given the equation of the irregular curve of stream, $y = 16x^2 \sin(x)$. Approximate the stream cross-sectional area of irregular shapes from $x = 0$ to $x = \frac{\pi}{2}$ into 5 equal intervals by using accurate Simpson's rule and express the absolute error. Do all calculation in 3 decimal places.

(10 marks)

- Q3** (a) The force on a yacht mast can be presented by the following function:

$$F = \int_0^H 200 \left(\frac{z}{7+z} \right) e^{-2z/H} dz$$

where z is the elevation above the deck and H is the height of the mast. By considering $H = 30 + m$, which m is referring to the last digit of your matrix number (example: AFxxxxx*m*), compute F using 2-point and 3-point Gauss quadrature. Do all calculation in 4 decimal places.

(13 marks)

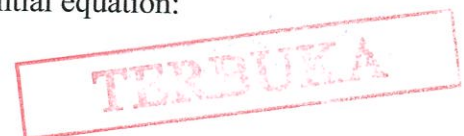
- (b) Given matrix T :

$$T = \begin{pmatrix} 3 & 2 & 3 \\ 2 & 6 & 6 \\ 3 & 6 & t \end{pmatrix}; \text{ where } t = [(last\ number\ of\ your\ Matric\ No.) + 2]$$

- (i) Find characteristic equation for matrix T
- (3 marks)
- (ii) Determine the dominant eigenvalue, and its corresponding eigenvectors using the Power Method. Let $\mathbf{v}^{(0)} = (1 \ 1 \ 1)^T$ and stop the iteration until $|m_{k+1} - m_k| \leq 0.0005$ or after five iterations, whichever comes first. Do all calculations in 4 decimal places.

(9 marks)

- Q4** (a) The problem of noise pollution in Kuala Lumpur has become more serious due to the development of transportation systems, construction, and industrial activities. Malaysia's Department of Environment (DOE) had imposed a noise limit of 60 decibels (dB) to protect the population from this urban noise. The noise level in Kuala Lumpur is given by the following differential equation:



$$\frac{dL}{dt} = \frac{1}{0.001 \cdot a \cdot \ln(10) \cdot L}$$

where L is the sound pressure level in decibel (dB) and a is the last digit of your matric number (for example: AFxxxxxa). If the last digit of your number is zero (0) then take $a = 1$.

- (i) Estimate the noise level in Kuala Lumpur from year 2022 to 2030 by using the fourth order Runge Kutta method with the increment of 2 years. Assume that the noise level in 2022 is 50.0 dB. Do all calculations in 3 decimal places.

(10 marks)

- (ii) Based on the answer in Q4 (a), are the noise levels acceptable in future built environment of Kuala Lumpur? Justify your answer.

(2 marks)

- (b) Given the partial differential equation (PDE) that use for bearing capacity resistance of 10m pile foundation in 10 minutes is

$$\frac{\partial^2 u}{\partial t^2} - 6 \frac{\partial^2 u}{\partial x^2} = 0$$

- (i) Based on the criteria of variables and partial derivatives, justify the category of the above PDE.

(3 marks)

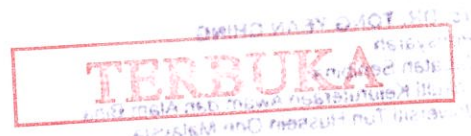
- (ii) Using the above equation for $0 < x < 10$, $0 < t < 10$, with boundary conditions, $u(0, t) = u(10, t) = 0$ for $0 \leq t \leq 10$, estimate the bearing capacity of the pile by using the explicit difference method.

Assume that the initial conditions $u(x, 0) = \sin\pi x$, $\frac{\partial u(x, 0)}{\partial t} = \sin 4\pi x$ for $0 \leq x \leq 10$, $h = \Delta x = 2.5\text{m}$ and $k = \Delta t = 5$ minutes.

Do all calculations in 3 decimal places.

(10 marks)

- END OF QUESTIONS -



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Table Q1(b): Polynomial for steel beam deflection.

x	0	0.25	0.75	1.0
$f(x) = e^{-x} + a$				

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FORMULAS

System of linear equations

Gauss-Seidel iteration : $x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)}}{a_{ii}}$

Interpolations

Newton's divided difference : $f_i^{[0]} = f_i, i = 0, 1, \dots, n$

$$f_i^{[j]} = \frac{f_{i+1}^{[j-1]} - f_i^{[j-1]}}{x_{i+1} - x_i}, j = 1, 2, \dots, n$$

$$f(x) \approx P_n(x) = f_0^{[0]} + f_0^{[1]}(x - x_0) + f_0^{[2]}(x - x_0)(x - x_1) + \dots + f_0^{[n]}(x - x_0)(x - x_1) \dots (x - x_{n-1})$$

Numerical Differentiation

First derivative,

2-point forward difference : $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2 - point backward difference : $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3 - point central difference : $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3 - point forward difference : $f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

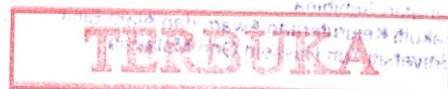
3 - point backward difference : $f'(x) \approx \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h}$

5 - point central difference : $f'(x) \approx \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)}{12h}$

Second derivatives,

3 - point central difference : $f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$

5 - point central difference : $f''(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h)}{12h^2}$



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Numerical integration

Simpson's $\frac{1}{3}$ rule : $A_1 = \int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2)$

$$\int_{b=x_n}^{b=x_n} f(x)dx \approx \frac{h}{3} \left(f_0 + f_n + 4 \sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f_i + 2 \sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f_i \right)$$

Simpson's $\frac{3}{8}$ rule : $A_1 = \int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3)$

$$\int_{b=x_n}^{b=x_n} f(x)dx \approx \frac{3h}{8} \left(f_0 + f_n + 3 \sum_{i=1}^{n/3} (f_{3i-2} + f_{3i-1}) + 2 \sum_{i=1}^{\left(\frac{n}{3}\right)-1} f_{3i} \right)$$

2-point Gauss Quadrature : $\int_{-1}^1 g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right) \right]$

3-point Gauss Quadrature : $\int_{-1}^1 g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right) \right]$

Eigenvalues

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}} Av^{(k)}, k = 0, 1, 2 \dots$

Ordinary-differential Equations (ODEs)

Fourth-Order Runge-Kutta Method (RK4) : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where

$$k_1 = hf(x_i, y_i) \qquad k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right) \qquad k_4 = hf(x_i + h, y_i + k_3)$$

Partial Differential Equations (PDEs)

Second Order Linear PDEs : $A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + Fu = G$

Wave Equation : $\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j}$

$$\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} \right)$$

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