

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

# FINAL EXAMINATION SEMESTER II SESSION 2021/2022

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**COURSE NAME** 

FOUNDATION ENGINEERING

COURSE CODE

BFC 43103

PROGRAMME CODE :

BFF

EXAMINATION DATE :

JULY 2022

**DURATION** 

3 HOURS

INSTRUCTION

1. ANSWER ALL QUESTIONS IN PART A AND TWO (2) QUESTIONS IN PART B.

- 2. THIS FINAL EXAMINATION IS AN ONLINE ASSESSMENT AND CONDUCTED VIA CLOSED BOOK.
- 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF SIXTEEN (16) PAGES

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TEMPERA

#### PART A

- Q1 (a) Identify how piles are classified according to the method of installation and function.

  (5 marks)
  - (b) The variation of  $N_{60}$  with depth in a granular soil deposit is as shown in **Table Q1(b)**. A rectangular concrete pile 7.5 m long (0.5 m x 0.5 m in cross section) is driven into the sand and fully embedded in the soil. For the proposed site, given: Design load = 1112.1 kN, FOS = 3.0,  $P_a = 100 \text{ kN/m}^2$  and  $L_{pile} = 7.5 \text{ m}$ .
    - (i) Design Qp by using data given in Table Q1(b)

(6 marks)

(ii) Design Q<sub>s</sub> by using data given in Table Q1(b)

(5 marks)

(iii) Estimate the allowable bearing capacity  $Q_{all}$  of the pile

(3 marks)

(iv) Predict the elastic settlement of the pile based on the situation.

(6 marks)

Q2 (a) The backfill used in the construction of the retaining wall has expansive behaviour. These materials swell significantly when in contact with water during rainfall and floods, resulting in large lateral pressure on the retaining wall. As a geotechnical engineer, propose TWO (2) methods to maintain the stability of the retaining wall.

(6 marks)

- (b) The proposed design of the cantilever retaining wall will be constructed on clay soil as shown in **Figure Q2** (b). The backfill is a coarse grained soil and inclined at 8°. The live load from a rural main road is 15 kN/m². The ground water level is located 2 m below the base.
  - (i) Evaluate the stability of the retaining wall in terms of overturning

(6 marks)

(ii) Design the stability of the retaining wall in terms of bearing capacity.

(5 marks)

(iii) Evaluate the factor of safety against sliding of the proposed retaining structure.

(5 marks)

(iv) If the factor of safety against sliding is less than 1.5, propose **THREE** (3) suitable methods to increase the stability against sliding.

(3 marks)

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TERBUKA

#### PART B

- Q3 (a) Due to high loading of traffic, the local government is planning to widen the federal road from Batu Pahat to Air Hitam in the near future. The Design Department of JKR is requested to propose ground improvement works that needs to be carried out in advance before commencement of the road widening project.
  - (i) The use of PVD together with surcharge load was proposed by the designer. Explain the working principles and details of this ground improvement technique with the illustration of relevant sketches.

(6 marks)

(ii) Discuss any TWO (2) disadvantages of using this technique when compared to soil replacement method.

(4 marks)

(b) Cement stabilization was proposed by the designer. Briefly discuss any TWO (2) advantages and TWO (2) disadvantages compared to the mechanical stabilization method using roller.

(8 marks)

(c) Evaluate whether dynamic compaction using tamper is suitable in this case. Based on the desk study, the soil formation at the proposed site is comprised of quaternary marine deposit.

(7 marks)

Q4 (a) A new building will be constructed at Taman Seri Indah area. For foundation, the consultant engineer has suggested to use a shallow foundation instead of deep foundation. Compare the difference between shallow foundation and deep foundation. Use sketches to support your answer.

(5 marks)

(b) The consultant engineer suggested to build a bungalow house with one floor at Elmina area. A 2.0 m wide strip foundation is placed in sand at 1.0 m depth. The properties of the sand are:  $\gamma = 19.5 \text{ kN/m}^3$ , c = 0, and  $\phi' = 34^0$ . Determine the maximum wall load that the foundation can carry, with a factor of safety of 3.0.

(8 marks)

(c) You anticipate the foundation for a proposed highway bridge will consist of end bearing piles driven through the overlying soils to bedrock. However, you are not certain the location of the bedrock. You wish to obtain at least an approximate idea of the depth to bedrock before you begin drilling the exploratory borings. Propose plan of work to obtain the data needed

(12 marks)



- Q5 (a) The road between Ayer Hitam and Batu Pahat had to be restricted due to the collapse of a 9-meter high soil slope. Mr. Ahmad recommended the cantilever sheet pile retaining structure as a method of stabilising the failing soil slope. Based on your experience,
  - (i) Write comment on Mr. Ahmad's selection of a retaining structure idea.

(4 marks)

(ii) Propose the suitable type of retaining structure, with a diagram to ensure the stability of the supported soil.

(5 marks)

(iii) Discuss the factors that should be considered while selecting a retaining structure.

(3 marks)

(b) (i) Explain in detail with the aid of sketches on what condition does pile foundations need to be used.

(5 marks)

(ii) A 12 m prestressed concrete pile 450 mm square is installed in a clay with water table at 5 m depth as shown in **Figure Q5(b)**. The upper clay layer is 5 m thick, with a unit weight (γ) of 17.4 kN/m³. The lower clay layer has a unit weight (γ) of 18.1 kN/m³. Determine the allowable pile capacity (Q<sub>all</sub>) with factor of safety of 2 by using the simplest method.

(8 marks)

-END OF QUESTIONS-



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Table Q1(b): The SPT value for the proposed site

Depth of Soil Layer	the strivate for the proposed site	
(m)	Description	SPT (N60)
0.0 - 1.50	Top soil, dark brown clayey silty sand	11
1.50 - 1.95	Medium dense with traces of gravel	11
3.00 - 3.45	Light greyish brown, silty sand	47
	Greyish brown silty sand with some	
4.50 – 4.95	gravel	8
6.00 - 6.45	Medium dense silty sand	19
	Medium grey silty sand with some	
7.50 - 7.95	gravel	31
9.00 - 9.33	Very dense silty sand	50
10.50 - 10.82	Very dense silty sand	50
12.00 - 12.32	Very dense silty sand	50
13.50 - 13.79	Very dense silty sand	50
15.00 - 15.445	Very dense silty sand	50
16.50 - 16.82	Very dense silty sand	50
18.00 - 18.32	Very dense silty sand	50



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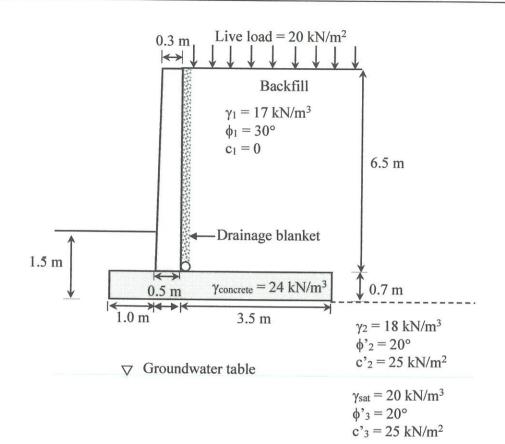


FIGURE Q2(b): Cantilever gravity retaining wall



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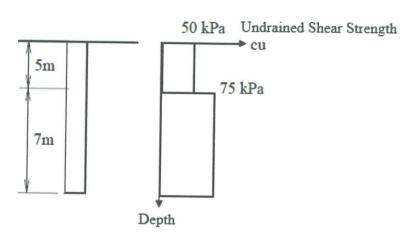


FIGURE Q5 (b): Undrained shear strength



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TABLE 1: Terzaghi Bearing Capacity factor

φ'	N <sub>c</sub>	$N_q$	N <sub>y</sub> *	φ'	N <sub>e</sub>	$N_q$	N <sub>γ</sub> *
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

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TABLE 2: Meyerhof's bearing capacity factor

φ'	N <sub>c</sub>	N <sub>q</sub>	N <sub>y</sub>	$\phi'$	N <sub>c</sub>	$N_q$	$N_{\gamma}$
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

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TABLE 3: Variation of α

α		
1.00		
0.92		
0.82		
0.74		
0.62		
0.54		
0.48		
0.42		
0.40		
0.38		
0.36		
0.35		
0.34		
0.34		

Note:  $p_a$  = atmospheric pressure  $\approx 100 \text{ kN/m}^2 \text{ or } 2000 \text{ lb/ft}^2$ 



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#### List of formula

#### SOIL IMPROVEMENT AND GROUND MODIFICATION

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma_o' + \Delta \sigma_{(p)}'}{\sigma_o'}$$

$$S_{c(p+f)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma_o^{'} + \left[\Delta \sigma_{(p)}^{'} + \Delta \sigma_{(f)}^{'}\right]}{\sigma_o^{'}}$$

$$U = \frac{\log \left[\frac{\sigma_{o}^{'} + \Delta \sigma_{(p)}^{'}}{\sigma_{o}^{'}}\right]}{\log \left[\frac{\sigma_{o}^{'} + \Delta \sigma_{(p)}^{'} + \Delta \sigma_{(f)}^{'}}{\sigma_{o}^{'}}\right]}$$

$$T_{v} = \frac{c_{v}t}{H_{c}^{2}}$$

For U%: 0% to 60%;  $T_{\nu} = \frac{\pi}{4} \left( \frac{U\%}{100} \right)^2$ 

For U% > 60%:

 $T_v = 1.781 - 0.931 \log (100 - U\%)$ 

$$U = \frac{\log \left[1 + \frac{\Delta \dot{\sigma_{(p)}}}{\dot{\sigma_{o}}}\right]}{\log \left[1 + \frac{\Delta \dot{\sigma_{(p)}}}{\dot{\sigma_{o}}} \left(1 + \frac{\Delta \dot{\sigma_{(p)}}}{\dot{\sigma_{(p)}}}\right)\right]}$$

 $N_{corrected} = C_N * N_{field}$ 

#### SITE INVESTIGATION

$$A_{R}(\%) = \frac{D_{o}^{2} - D_{i}^{2}}{D_{i}^{2}}(\%)$$

$$C_N = 0.77 \log_{10} \frac{1915}{p'_o}$$

$$N_{60} = \frac{N\eta_H \eta_B \eta_S \eta_R}{60}$$

where

N<sub>60</sub> = Standard penetration number, corrected for field conditions.

 $\eta_H = \text{Hammer Effiency (\%)}$ 

 $\eta_B$  = Correction for borehole diameter

 $\eta_S =$ Sampler correction

 $\eta_R$  = Correction for rod length

Variation of nB

Diameter (mm)	$\eta_{\rm B}$
60 - 120	1
150	1.05
200	1.15

Variation of \( \eta\_S \)

Rod lenght (mm)	$\eta_{\rm B}$
Standard sampler	1.0
With liner for dense sand and clay	0.8
With liner for loose sand	0.9

Schmertmann's (1975) theory

$$\phi = tan^{-1} \left[ \frac{N_{60}}{12.2 + 20.3 \left( \frac{\sigma_0'}{P_a} \right)} \right]^{0.34}$$

where,  $\sigma_0'$  = effective overburden pressure

 $(kPa) = \gamma H$ 

 $P_a = atmospheric pressure$ 

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#### SHALLOW FOUNDATIONS

## Modification of Bearing Capacity Equations for Water Table

Case I for water within 0 <

$$D_1 \leq D_f$$
;  
 $D_1 \leq D_f$ ;  
 $D_1 \leq D_f$ ;

$$q = D_1 \gamma_{dry} + D_2 \left( \gamma_{sat} - \gamma_w \right)$$
$$\gamma' = \gamma_{sat} - \gamma_w$$

Case II for water within 0 < d <

$$q = D_1 \gamma_{dry}$$

$$\overline{\gamma} = \gamma' + \frac{d}{B} \left( \gamma_{dry} - \gamma' \right)$$

Case III when the water table is located so that  $d \ge B$ , the water will have no effect on the ultimate bearing capacity.

$$q_{u} = c' N_{c} F_{cs} F_{cd} F_{ci} + q N_{q} F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_{\gamma} F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

### Shape Factor

$$F_{cs} = 1 + \frac{B}{L} \cdot \frac{N_q}{N_c}$$

$$F_{qs} = 1 + \frac{B}{L} \tan \phi$$

$$F_{\gamma s} = 1 - 0.4 \frac{B}{L}$$

#### Depth Factor

$$D_f/B \le 1, for \phi = 0$$

$$F_{cd} = 1 + 0.4 \binom{D_f}{B}$$

$$F_{qd} = 1$$

$$F_{\gamma d}=1$$

$$D_f/B \le 1, for \ \phi > 0$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c tan \phi'}$$

$$F_{qd} = 1 + 2tan\phi'(1 - sin\phi')^2 \frac{D_f}{B}$$

$$F_{\gamma d}=1$$

$$F_{cd} = 1 + \underbrace{0.4 \, tan^{-1} \binom{D_f}{f}}_{\text{radians}}$$

$$\frac{D_f/B > 1, for \ \phi = 0}{F_{qd} = 1}$$

$$F_{\gamma d} = 1$$

$$D_f/B > 1$$
, for  $\phi > 0$ 

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c tan\phi'}$$

$$F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 tan^{-1} (\frac{D_f}{B})$$

$$F_{\gamma d}=1$$

where L is the length of the foundation and L>B.

#### **Inclination Factor**

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^{\circ}}{90^{\circ}}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)^2$$

 $\beta$  is the inclination of the load on the foundation with respect to vertical

Eccentric Loading in Shallow Foundations



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#### SHALLOW FOUNDATIONS

$$q'_{u} = c' N_{c} F_{cs} F_{cd} F_{ci} + q N_{q} F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma' B' N_{\gamma} F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

# One Way Eccentric Loading in Shallow Foundations

Method 1:

$$B' = B - 2e$$

$$L' = L$$

$$q'_{u} = c'N_{c}F_{cs}F_{cd}F_{ci} + qN_{q}F_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma'B'N_{\gamma}F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$Q_{ult} = q'_{u}B'L$$

$$Q_{ult} = B \left[ c' N_{c(e)} + q N_{q(e)} + \frac{1}{2} \gamma B N_{\gamma(e)} \right]$$

$$Q_{ult} = BL \left[ c' N_{c(e)} F_{cs(e)} + q N_{q(e)} F_{qs(e)} + \frac{1}{2} \gamma B N_{\gamma(e)} F_{\gamma s(e)} \right]$$

$$F_{cs(e)} = 1.2 - 0.025 \frac{L}{R}$$

$$F_{as(e)} = 1.00$$

$$F_{\gamma s(e)} = 1.0 + \left(\frac{2e}{B} - 0.68\right) \frac{B}{L} + \left[0.43 - \left(\frac{3}{2}\right) \left(\frac{e}{B}\right)\right] \left(\frac{B}{L}\right)^2$$

Method 3:

$$R_k = 1 - \frac{q_{u(eccentric)}}{q_{u(centric)}}$$

$$q_{u(centric)}$$

$$R_k = a \left(\frac{e}{B}\right)^k$$

$$q_{u(eccentric)} = q_{u(centric)} (1 - R_k)$$

$$q_{u(centric)} = qN_qF_{qd} + \frac{1}{2}\gamma BN_{\gamma}F_{\gamma d}$$

 $Q_{ult} = Bq_{u(eccentric)}$ 

# Primary Consolidation Settlement for Normally Consolidated Clays

$$S_{c(p)} = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma_o' + \Delta \sigma_{av}'}{\sigma_o'}, \text{ for 2:1 method } \Delta \sigma_{(1)}' = \frac{Q_g}{(L_g + z_1)(B_g + z_1)}$$

# Primary Consolidation Settlement for OverConsolidated Clays

for 
$$\sigma'_o + \Delta \sigma'_{av} < \sigma'_c$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta \sigma'_{av}}{\sigma'_o}$$

for 
$$\sigma'_o < \sigma'_c < \sigma'_o + \Delta \sigma'_{av}$$

$$S_{c(p)} = \frac{C_s H_c}{1 + e_o} \log \frac{\sigma'_c}{\sigma'_o} + \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta \sigma'_{av}}{\sigma'_o}$$

#### Average Increase in Pressure

$$\Delta\sigma_{av}' = rac{1}{6} \left( \Delta\sigma_{top}' + 4\Delta\sigma_{medium}' + \Delta\sigma_{bottom}' \right), \Delta\sigma_{top/middle/bottm}' = q_o I_c$$
 $m_1 = rac{L}{B}, n_1 = rac{Z}{(B/2)}$ 

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A LILLO WALLEY

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#### LATERAL EARTH PRESSURE AND RETAINING WALLS

$$K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$L_3 = \frac{\sigma_2'}{\gamma'(K_p - K_a)}$$

$$\sigma_{5}' = (\gamma L_{1} + \gamma' L_{2}) K_{p} + \gamma' L_{3} (K_{p} - K_{a})$$

$$A_{1} = \frac{\sigma_{5}'}{\gamma' (K_{p} - K_{a})}$$

$$A_1 = \frac{\sigma_5'}{\gamma' \left(K_p - K_a\right)}$$

$$A_2 = \frac{8P}{\gamma'(K_0 - K_1)}$$

$$A_3 = \frac{6P\left[2\bar{z}\gamma'\left(K_p - K_a\right) + \sigma_5'\right]}{\gamma'^2\left(K_p - K_a\right)^2}$$

$$A_4 = \frac{P\left[6z\sigma_5' + 4P\right]}{\gamma'^2 \left(K_o - K_a\right)^2}$$

$$L_4^4 + A_1L_4^3 - A_2L_4^2 - A_3L_4 - A_4 = 0$$

$$\sigma_4' = \sigma_5' + \gamma' L_4 (K_p - K_a)$$

$$\sigma_3' = L_4 \left( K_p - K_a \right) \gamma'$$

$$L_5 = \frac{\sigma_3' L_4 - 2P}{\sigma_3' + \sigma_4'}$$

$$K_{p(design)} = \frac{K_{p}}{FS}$$

$$S_V = \frac{T_{\text{all}}}{\gamma_1 z K_a [FS_{(B)}]}$$

$$L = \frac{H - z}{\tan\left(45 + \frac{\phi_1'}{2}\right)} + \frac{S_V K_a[FS_{(B)}]}{2\tan\phi_F'}$$

$$l_l = \frac{S_V K_a [FS_{(P)}]}{4 \tan \phi_T'}$$

$$v = \sqrt{\frac{E}{\left(\frac{\gamma}{g}\right)} \frac{(1-\mu)}{(1-2\mu)(1+\mu)}}$$

$$Z_{1} = \frac{1}{2} \sqrt{\frac{v_{2} - v_{1}}{v_{2} + v_{1}}} x_{c}$$

$$z_{2} = \frac{1}{2} \left[ T_{i2} - 2 Z_{1} \frac{\sqrt{v_{3}^{2} - v_{1}^{2}}}{v_{3} v_{1}} \right] \frac{v_{3} v_{2}}{\sqrt{v_{3}^{2} - v_{2}^{2}}}$$