Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	CALCULUS
COURSE CODE	:	BFC 15003
PROGRAMME	:	BFF
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTIONS	1.	ANSWER ALL QUESTIONS
	2.	THIS FINAL EXAMINATION IS **AN ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSE BOOK**
	3.	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Determine the derivative of $g(t) = \tan(8 + \sin 4t)$

(4 marks)

(b) An equation is given as $3y^3 = 3x^2 + \sin xy$. Determine the first derivative of the equation in terms of x

(5 marks)

(c) By using a suitable method, determine the first and second derivatives of

$$f(t) = \frac{t^2 - 1}{t^2 + t - 2}$$

(6 marks)

(d) In an experiment, a concrete block as shown in **Figure Q1 (d)**, is subjected to constant heat for approximately three hours. It is observed that, the length of the concrete block increase at a constant rate of 0.5mm/hours.

(i) State the total surface area of the concrete block, A in terms of x and find $\frac{dA}{dx}$

(2 marks)

(ii) If the initial length and width of the concrete block are 300 mm and 100 mm respectively. Examine the length and width of the block after being heated for 40 minutes.

(4 marks)

(iii) Evaluate the rate of increase in the surface area at that time

(4 marks)

Q2 (a) An equation of a curve is given as $y = 2x^3 + 5x - 8$. Identify the slopes of the tangent lines to the curve at (-1,-15), (0, -8), (1,-1) and (2,18).

(5 marks)

(b) A function of $g(x) = x^3 + Px^2 + Qx + R$ has critical numbers at $x = -1$ and $x = 3$. Determine the constant values of P and Q

(5 marks)

(c) Given a function of a curve is $\frac{2}{3}x^3 - 9x^2 + 36x + 2$,

(i) Determine the critical numbers of the curve

(3 marks)

(ii) By using the Second Derivative Test, determine the local maximum and minimum point of the function and identify the shape of the graph.

(10 marks)

(iii) Sketch a curve that shows the local maximum and minimum points

(2 marks)

Q3 (a) Prove the following definite integral

$$\int_1^e 2(xlnx)\, dx = \frac{1}{2}(e^2 + 1)$$

(5 marks)

(b) The following indefinite integral shows that

$$\int \frac{1+2x}{(x-3)(x-9)^2} dx = \int \frac{P}{(x-3)} dx + \int \frac{Q}{(x-9)} dx + \int \frac{R}{(x-9)^2} dx$$

(i) Determine the values of constant P, Q, R

(4 marks)

(ii) Prove that
$\int \frac{1+2x}{(x-3)(x-9)^2} dx = \frac{7}{36}\ln(x-3) - \frac{7}{36}\ln(x-9) - \frac{19}{6(x-9)} + C$,

Hence identify the integral with 12 and 15 as the lower and upper limit respectively.

(3 marks)

(c) Sketch a curves and determine the area of the region enclosed by the parabola curve $y = x^2 - 8$ and $y = -2x$

(8 marks)

(d) Determine the solution of $\frac{dy}{dx} + y\tan x - \cos x = 0$. Given that $y = 1$ and $x = 0$

(5 marks)

Q4 (a) **Figure Q4 (a)** shows a steel bar subjected to the external forces Q and P. The differential equation of the steel bar is given as

$$EA\frac{d^2y}{dx^2} = -Q$$

with boundary conditions of $EA\frac{dy}{dx} = P$ at $x = \text{L}$ and $y(x) = 0$ at $x = 0$.

Given E is the young modulus of the steel bar, A is the cross-sectional, L is the length of the bar and $y(x)$ is the axial displacement.

(i) Determine the solution of the axial displacement

(7 marks)

(ii) Given:

$Q = 5$ kN, $P = 12$ kN, L = 1.5 m, $E = 210000\ N/mm^2$ and $A = 2500\ mm^2$, evaluate the axial displacement at the middle of the bar.

(3 marks)

(b) A tennis ball that weighs about 0.056kg is hit straight up with an initial velocity of 1.37m/s. Air resistance acts on the ball with a force of 0.5v, where v represents the ball's velocity at time t.

(i) Determine the velocity of the ball as a function of time

(9 marks)

(ii) How to indicate that the ball has reached its maximum height. Explain the reason.

(3 marks)

(iii) How long does it take for the ball to reach its maximum height?

(3 marks)

- END OF QUESTIONS -

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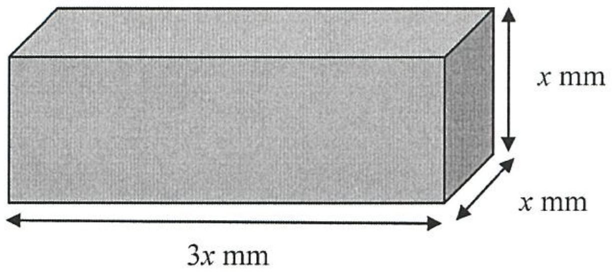

Figure Q1 (d)

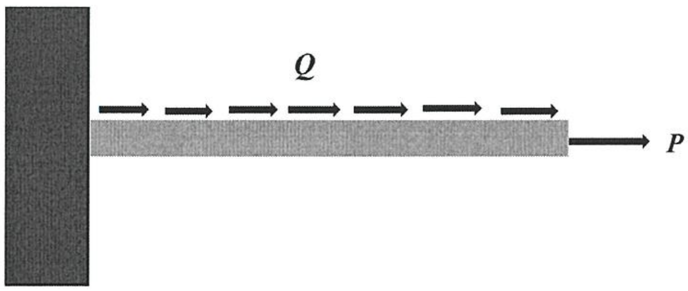

Figure Q4 (a)

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Formula

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C \qquad n \neq -1$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x \tan x$	$\int \sec x \tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x \cot x$	$\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x \tanh x$	$\int \text{sech}\,x \tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x \coth x$	$\int \text{cosech}\,x \coth x\,dx = -\text{cosech}\,x + C$

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Formula

Trigonometric	Hyperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \csc^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	
	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x
$a^x = e^{x \ln a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \geq 1$
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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Formula

Integration of Inverse Function
$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\lvert a\rvert\sqrt{a^2-x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$
$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+C, \quad a>0$
$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+C, \quad x>a$
$\int \frac{dx}{x^2-a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert<a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & \lvert x\rvert>a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$
$\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right)+C, \quad 0<x<a$

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Formula

Differentiation of Inverse Functions	
y	$\dfrac{dy}{dx}$
$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u \rvert < 1$
$\cos^{-1} u$	$-\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u \rvert < 1$
$\tan^{-1} u$	$\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\cot^{-1} u$	$-\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\sec^{-1} u$	$\dfrac{1}{\lvert u \rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u \rvert > 1$
$\operatorname{cosec}^{-1} u$	$-\dfrac{1}{\lvert u \rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u \rvert > 1$
$\sinh^{-1} u$	$\dfrac{1}{\sqrt{u^2+1}}\dfrac{du}{dx}$
$\cosh^{-1} u$	$\dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u \rvert > 1$
$\tanh^{-1} u$	$\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u \rvert < 1$
$\coth^{-1} u$	$-\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u \rvert > 1$
$\operatorname{sech}^{-1} u$	$-\dfrac{1}{u\sqrt{1-u^2}}\dfrac{du}{dx}, \quad 0 < u < 1$
$\operatorname{cosech}^{-1} u$	$-\dfrac{1}{\lvert u \rvert\sqrt{1+u^2}}\dfrac{du}{dx}, \quad u \neq 0$