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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	ACTUARIAL MATHEMATICS II
COURSE CODE	:	BWA 31503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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Q1 (a) Cash value is the amount available in cash upon cancellation of an insurance policy.

(i) Identify **THREE (3)** common insurance options that use the policy's net cash values.

(3 marks)

(ii) Discuss the similarities or differences among these three insurance options given in **Q1(a)(i)**.

(7 marks)

(b) You are given the Illustrative Life Table **(Table Q1(b))** with interest rate of 6%. A 40-year-old male purchased the 10-year endowment insurance where the net level premium is RM 81.36 and the death benefit is RM 1,000. Determine the terminal reserve for the fourth year of a 40-year old male.

Table Q1(b)

Age	Number Alive	Number That Die
40	937.72	2.83
41	934.89	3.08
42	931.81	3.32
43	928.50	3.59
44	924.90	3.88

(10 marks)

Q2 (a) Consider an insurance portfolio that will produce zero, one, two, or three claims in a fixed time period with probabilities 0.1, 0.3, 0.4, and 0.2, respectively. An individual claim will be of amount 1, 2, or 3 with probabilities 0.5, 0.4, and 0.1 respectively.

(i) Construct a table and compute $f_s(x) = \Pr(S = s)$ for $x = 0,1,2,3,4$.

(10 marks)

(ii) Find $E[N]$, $Var(N)$ and $E[X]$.

(4 marks)

(b) Suppose that the claim amount distribution is the same as in **Q2(a).** The distribution of N follows Poisson distribution. The formula for the expectation of S given by

$$E[S] = \lambda E[X] = \lambda p_1,$$

and the variance of S

$$Var(S) = \lambda E[X^2] = \lambda p_2.$$

Use these formulas and $E[N]$ from **Q2(a)(ii)** to compute $E[S]$ and $Var(S)$.

(6 marks)

Q3 (a) The actuarial present value of the family income benefit is given by

$$E(Z)=\int_0^n v^t \bar{a}_{\overline{n-t|}}\ {}_tp_x\ \mu_x(t)\,dt$$

This integral can be converted to a current payment integral by integration by parts,

$$\bar{a}_{\overline{n|}}-\int_0^n v^t\ {}_tp_x dt=\int_0^n v^t\left(1-{}_xp_x\right)dt=\bar{a}_{\overline{n|}}-\bar{a}_{x:\overline{n|}}$$

A policy provides a continuous annuity-certain of 1 per annum beginning at the date of death of age 35. You are given the following information:

- In the event of death prior to age 65, a family income benefit ceasing at age 65, and
- In the event of survival to age 65, a life annuity with 5 years certain.

(i) Construct a table that gives the conditions required for payments at time t and the corresponding probabilities.

(6 marks)

(ii) Calculate the actuarial present value of the benefits.

(4 marks)

(b) Consider a policy issued at age 35 with an initial gross premium of 1,000 and an initial benefit of 120,000. The policyholder wishes to change the premium of the policy after 5 years to 1,500 and the benefit amount to 150,000. Use the Illustrative Life Table (**Table Q3(b)**) with 6% interest to determine the reserve after 10 years of original issue if the fifth year reserve is 3,321.25.

Table Q3(b)

Age	l_x	d_x	$1,000q_x$
30	95 013.79	145.2682	1.5289
31	94 868.53	152.6317	1.6089
32	94 715.89	160.6896	1.6965
33	94 555.20	169.5052	1.7927
34	94 385.70	179.1475	1.8980
35	94 206.55	189.6914	2.0136
36	94 016.86	201.2179	2.1402
37	93 815.64	213.8149	2.2791
38	93 601.83	227.5775	2.4313
39	93 374.25	242.6085	2.5982
40	93 131.64	259.0186	2.7812
41	92 872.62	276.9271	2.9818
42	92 595.70	296.4623	3.2017
43	92 299.23	317.7619	3.4427
44	91 981.47	340.9730	3.7070
45	91 640.50	366.2529	3.9966

(10 marks)

Q4 (a) Ahmad's age is 50.5 at valuation date. He receives RM6,000 in salary in the month to the valuation date. Ahmad salary increases yearly on 1 January and he is planning to retire at age 65. Assume the replacement ratio is 65% and valuation date of 1 September. Using Hypothetical Salary Scale **(Table Q4(a)),**

Table Q4(a)

Age	s_x	x	s_x
30	1.00	50	3.41
31	1.06	51	3.63
32	1.13	52	3.86
33	1.20	53	4.10
34	1.28	54	4.35
35	1.36	55	4.62
36	1.44	56	4.91
37	1.54	57	5.21
38	1.63	58	5.53
39	1.74	59	5.86
40	1.85	60	6.21
41	1.96	61	6.56
42	20.9	62	6.93
43	2.22	63	7.31
44	2.36	64	7.70
45	2.51	65	8.08
46	2.67	66	8.48
47	2.84	67	8.91
48	3.02	68	9.35
49	3.21	69	9.82

(i) determine the salary that Ahmad receives over the year of age $\left(49\frac{5}{6}, 50\frac{5}{6}\right)$,

(1 mark)

(ii) calculate the expected salary in Ahmad final year of work,

(5 marks)

(iii) calculate Ahmad target pension benefit per year.

(3 marks)

(b) Suppose Fasha aged 30 is a newly hired employee of DRB Group. She receives RM80,000 in her first year of service at the company. Assuming

- Fasha salary increases 3% per year,
- she receives merit increases of 5% at each of the first three (3) employment anniversaries,
- the pension benefit formula is 1% of the final five (5) year average salary per year service.

(i) If Fasha retires at age 65, predict the projected final **FIVE (5)** year average salary.

(5 marks)

(ii) Forecast the projected pension benefit Fasha will receive at age 65.

(2 marks)

(iii) Compute the employee's replacement ratio.

(4 marks)

Q5 A Lexis diagram provides a convenient way of showing the relationship between periods and cohorts. Demographic events can be viewed either by calendar time, age or cohort.

(a) Using the Lexis diagram in **Figure Q5(a)**, calculate

(i) the average age of employees at time -25,

(2 marks)

(i) the number of employees who have attained age 35 while active in the workforce,

(1 mark)

(iii) the number of employees at time -25, who have attained or will attain age 50 while in the workforce.

(1 mark)

(b) The generation force of mortality at age x for those born at time u is denoted by

$$\mu(x,u) = -\frac{1}{l(x,u)}\frac{\partial}{\partial x}l(x,u).$$

From **Figure Q5(b)**, use the double integral method to show that the number of lives that will attain age x_0 between times t_0 and t_0+1 and die before time t_0+3 is given by

$$\int_{t_0}^{t_0+1} l(x_0, y-x_0)\,dy - \int_{x_0+2}^{x_0+3} l(w, t_0+3-w)\,dw.$$

(10 marks)

(c) A population density function is defined by

$$l(x,u) = b(u)s(x,u).$$

Let

$$b(u) = 100\left[1 - e^{-u/100}\right] \quad u > 0$$
$$s(x) = e^{-x/100} \quad x > 0.$$

Calculate the number of individuals between ages 25 and 50 at time 100.

(6 marks)

-END OF QUESTIONS-

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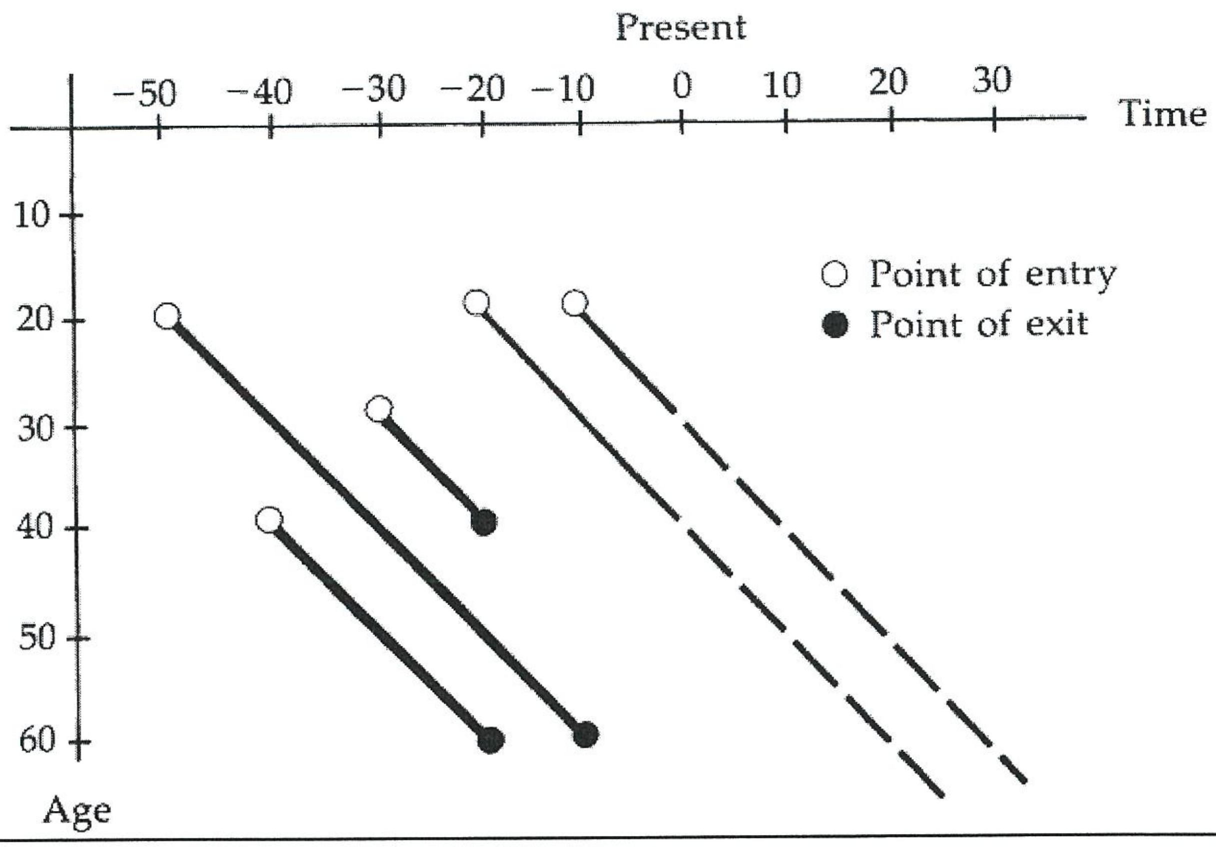

Figure Q5(a)

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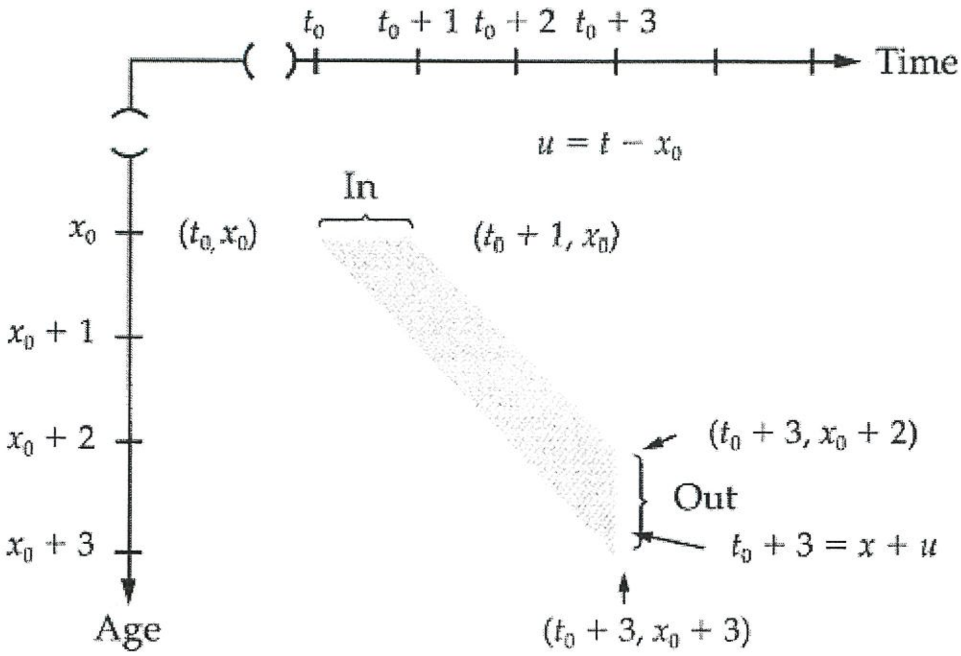

Figure Q5(b)

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FORMULAE

$$\Pr(N=n)=\frac{\lambda^n e^{-\lambda}}{n!}$$

$$\ddot{a}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^k \cdot {}_kp_x$$

$$A^1_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^{k+1}\cdot {}_kp_x\cdot q_{x+k}$$

$${}_nE_x = v^n\,{}_nP_x$$

$$v^n=\frac{1}{(1+i)^n}$$

$${}_kP_x=\frac{l_{40+k}}{l_{40}}$$

$$-{}_0V=P-\upsilon q_x b$$

$${}_kV=\frac{{}_0V+P\ddot{a}_{x:\overline{k}|}-bA^1_{x:\overline{k}|}}{{}_kE_x}$$

$${}_{k+g}V'=\frac{{}_kV'+P'\ddot{a}_{x+k:\overline{g}|}-b'A^1_{x+k:\overline{g}|}}{{}_gE_{x+k}}$$

$$\int_{t_0}^{t_1} l(x,t-x)\,dt$$

$$\int_{x_0}^{x_1} l(x,t_0-x)\,dx$$