UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BDU 11003
PROGRAMME CODE	:	BDC/BDM
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**. 2. THIS FINAL EXAMINATION IS A CONDUCTED **CLOSE BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x)=\begin{cases}0, & -\pi \le x < -0.5\pi \\ 1, & -0.5\pi < x < 0.5\pi \\ 0, & 0.5\pi < x \le \pi\end{cases}$$

and $f(x)=f(x+2\pi)$.

(a) Sketch the graph of $f(x)$ over $-3\pi < x < 3\pi$.

(2 marks)

(b) Find the Fourier coefficients corresponding to $f(x)$.

(13 marks)

(c) From (b), prove that the Fourier series for $f(x)$

$$f(x)=\frac{1}{2}+\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\cos(2n-1)x}{2n-1}.$$

(5 marks)

Q2 (a) The upward velocity of a rocket, measured at 3 different times, is shown in the **Table Q2(a)**. The velocity over the time interval $5 \le t \le 12$ is approximated by a quadratic expression as;

$$v(t)=a_1 t^2 + a_2 t + a_3 .$$

Determine the values of a_1, a_2 and a_3 by using Gauss – Elimination method.

(10 marks)

(b) Solve the system of linear equations by using Thomas algorithm method.

$$\begin{pmatrix}0 & 0 & 3 & 4\\ 2 & 9 & 1 & 0\\ 0 & 1 & 9 & 4\\ 2 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}a\\ b\\ c\\ d\end{pmatrix}=\begin{pmatrix}8\\ 6\\ 8\\ 9\end{pmatrix}$$

(10 marks)

PART B

Q3 (a) By using an appropriate method, solve

$$y'' - 2y' - 3y = 4e^{3x} + 9x$$

with $y(0) = 2$ and $y'(0) = -2$.

(13 marks)

(b) A mass of 20.4 kg is suspended from a spring with a known spring constant of 29.4 N/m. The mass is set in motion from its equilibrium position with an upward velocity of 3.6m/s. The motion can be described in the differential equation

$$\ddot{x} + \frac{k}{m}x = 0$$

where m is the mass of the object and k is the spring constant.

(i) Identify the initial conditions.
(ii) Find an equation for the position of the mass at any time t.

(7 marks)

Q4 (a) Determine the Laplace transform for each of the following function:

(i) $(2 + t^3)e^{-2t}$.
(ii) $\sin(t - 2\pi)\, H(t - 2\pi)$.
(iii) $\sin 3t\ \delta(t - \pi)$.

(10 marks)

(b) Consider the periodic function

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1 - t, & 1 \le t < 2 \end{cases},$$
$$f(t) = f(t + 2).$$

Sketch the graph of $f(t)$ and find its Laplace transform.

$$\left[\text{Hint: } \mathcal{L}[f(t)] = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t)\, dt, \quad s > 0.\right]$$

(10 marks)

Q5 (a) (i) Find the inverse Laplace transform of

$$\frac{s+3}{s^2-6s+13}$$

(ii) From **Q5(a)(i)**, find

$$\mathcal{L}^{-1}\left\{\frac{(s+3)e^{-\frac{1}{2}\pi s}}{s^2-6s+13}\right\}$$

(8 marks)

(b) (i) Express

$$\frac{1}{(s-1)(s-2)^2}$$

in partial fractions and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)^2}\right\} = e^t - e^{2t} + te^{2t}.$$

(ii) Use the result in **Q5(b)(i)** to solve the differential equation

$$y' - y = te^{2t}$$

which satisfies the initial condition of $y(0) = 1$.

(12 marks)

Q6 (a) (i) Show that $\frac{dy}{dx} = \frac{y}{y-x}$ is a homogeneous differential equation.

(ii) Hence, solve the differential equation in part **Q6(a)(i)**.

(12 marks)

(b) Choose an appropriate method to solve the following problem and obtain its particular solution

$$\frac{dy}{dx} - y\tan x = \sec x,\ y(0) = 1.$$

(8 marks)

-END OF QUESTIONS-

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Table Q2(a): Upward velocity of a rocket

Time , t (seconds)	Velocity, v (meters/second)
5	106.8
8	177.2
12	279.2

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Formula's

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) - A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n\frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt, \quad s>0.$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi x}{L} + b_n\sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$