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UTHM

Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

**FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022**

- COURSE NAME : ENGINEERING TECHNOLOGY
MATHEMATICS I
- COURSE CODE : BDU 10903
- PROGRAMME : BDC/ BDM
- EXAMINATION DATE : JULY 2022
- DURATION : 3 HOURS
- INSTRUCTION : 1. ANSWER FIVE (5) QUESTIONS
ONLY
2. THIS FINAL EXAMINATION IS
CONDUCTED VIA **CLOSED BOOK**
3. STUDENTS ARE **PROHIBITED** TO
CONSULT THEIR OWN MATERIAL
OR ANY EXTERNAL RESOURCES
DURING THE EXAMINATION
CONDUCTED VIA **CLOSED BOOK**

THIS QUESTION PAPER CONSISTS OF SEVEN (7) PAGES

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Q1 Given a helix, $\underline{r}(t) = 4(\cos t) \underline{i} + 4(\sin t) \underline{j} + t \underline{k}$, $0 \leq t \leq 4\pi$.

- (a) Sketch the helix. (6 marks)
- (b) Find the unit tangent vector at t . (5 marks)
- (c) Find the curvature for $\underline{r}(t)$. (5 marks)
- (d) Find the arc length for $\underline{r}(t)$. (4 marks)

Q2 (a) Find the Maclaurin polynomials p_0, p_1, p_2, p_3 and p_n for $f(x) = \cos x$. (7 marks)

(b) Find the radius of convergence of $\sum_{n=0}^{\infty} 3(x-2)^n$. (6 marks)

(c) Find a power series for $f(x) = \ln x$, centered at 1. (7 marks)

Q3 (a) Find the limits

(i) $\lim_{x \rightarrow e^2} \frac{(\ln x)^3 - 8}{\ln x - 2}$ (4 marks)

(ii) $\lim_{x \rightarrow 0^+} \frac{\sin x}{5\sqrt{x}}$ (3 marks)

(iii) $\lim_{x \rightarrow \infty} \frac{(1 + 5x^{1/3} + 2x^{5/3})^2}{x^5}$ (4 marks)



- (b) Find constants A and B , so that the following function $f(x)$ will be continuous for all x .

$$f(x) = \begin{cases} \frac{x^2 - Ax - 6}{x - 2}, & x > 2 \\ x^2 + B, & x \leq 2. \end{cases}$$

(5 marks)

- (c) Express $\frac{(2+i)^2}{2-3i}$ in the form of $a+ib$.

(4 marks)

- Q4** (a) If $y = x + \cos(xy)$, find $\frac{dy}{dx}$.

(4 marks)

- (b) Find $\frac{dy}{dx}$ of $x = e^{-t} \cos 2t$ and $y = e^{-2t} \sin 2t$.

(6 marks)

- (c) Let $f(x) = \frac{x}{(x+1)^2}$.

(i) Show that $f(x)$ has a vertical asymptote at $x = -1$ and a horizontal asymptote at x -axis.

(ii) Find the critical point for $f(x)$. Determine the extremum points.

(iii) If $f(x)$ have inflection point at $x = 2$, sketch the graph of $f(x)$ and show the inflection point in that graph.

(10 marks)

- Q5** (a) Evaluate $\int \frac{dx}{1+9x^2}$.

(5 marks)

- (b) Use the trigonometry substitution to integrate $\int \frac{1}{x^2 \sqrt{1-x^2}} dx$.

(8 marks)

- (c) Find $\int (\sin 3x - \cos x)^2 dx$.

(7 marks)

- Q6** (a) The radius of a circle is increasing at the rate of 5 cm per second. Find
- (i) the rate of change of the area of the circle when its radius is 12 cm.
[Hint: Area, $A = \pi r^2$]
(4 marks)
 - (ii) the radius of the circle when its area is increasing at a rate of $50\pi \text{ cm}^2\text{s}^{-1}$.
(2 marks)
- (b) A particle P is moving along the x -axis, such that its displacement x at time t is $x(t) = t^2 - 4t$, where t is measured in seconds and $x(t)$ is measured in meters. Find the acceleration of the particle.
(2 marks)
- (c) Evaluate $\int \frac{x-3}{3x^2+2x-5} dx$ using partial fractions.
(12 marks)

- END OF QUESTIONS -

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Formulae

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x dx = \sinh x + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x dx = -\operatorname{coth} x + C$$

$$\int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \operatorname{coth} x dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \quad |x| < 1$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \quad |x| < 1$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \quad |x| > 1$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \quad |x| > 1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}} dx = \operatorname{sech}^{-1} |x| + C, \quad 0 < x < 1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}} dx = \operatorname{csch}^{-1} |x| + C, \quad x \neq 0$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & |x| < 1 \\ \operatorname{coth}^{-1} x + C, & |x| > 1 \end{cases}$$

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TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

TRIGONOMETRIC SUBSTITUTION

<i>Expression</i>	<i>Trigonometry</i>	<i>Hyperbolic</i>
$\sqrt{x^2 + k^2}$	$x = k \tan \theta$	$x = k \sinh \theta$
$\sqrt{x^2 - k^2}$	$x = k \sec \theta$	$x = k \cosh \theta$
$\sqrt{k^2 - x^2}$	$x = k \sin \theta$	$x = k \tanh \theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan \frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$	$dx = \frac{2dt}{1+t^2}$	$\tan 2x = \frac{2t}{1-t^2}$	$dx = \frac{dt}{1+t^2}$

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IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

<i>Trigonometric Functions</i>	<i>Hyperbolic Functions</i>
$\cos^2 x + \sin^2 x = 1$ $\sin 2x = 2 \sin x \cos x$ $\cos 2x = \cos^2 x - \sin^2 x$ $= 2 \cos^2 x - 1$ $= 1 - 2 \sin^2 x$ $1 + \tan^2 x = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$ $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ $\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$ $\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$ $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$ $2 \sin ax \cos bx = \sin(a + b)x + \sin(a - b)x$ $2 \sin ax \sin bx = \cos(a - b)x - \cos(a + b)x$ $2 \cos ax \cos bx = \cos(a - b)x + \cos(a + b)x$	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\cosh x = \frac{e^x + e^{-x}}{2}$ $\cosh^2 x - \sinh^2 x = 1$ $\sinh 2x = 2 \sinh x \cosh x$ $\cosh 2x = \cosh^2 x + \sinh^2 x$ $= 2 \cosh^2 x - 1$ $= 1 + 2 \sinh^2 x$ $1 - \tanh^2 x = \operatorname{sech}^2 x$ $\operatorname{coth}^2 x - 1 = \operatorname{csch}^2 x$ $\tanh 2x = \frac{2 \tanh x}{1 + \tanh^2 x}$ $\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$ $\sinh(x \pm y) = \sinh x \cosh y \pm \sinh y \cosh x$ $\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

CURVATURE, ARC LENGTH AND TANGENT VECTORS

$$\kappa = \frac{\|d\mathbf{T}/dt\|}{\|d\mathbf{r}/dt\|}$$

$$s(t) = \int_a^b \|r'(t)\| dt$$

$$\underline{T}(t) = \frac{r'(t)}{\|r'(t)\|}, \quad r'(t) \neq 0$$

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