UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	NUMERICAL METHODS
COURSE CODE	:	BDA 34103
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS IN **PART A** 2. ANSWER **ONE (1)** QUESTION IN **PART B** 3. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 **Table Q1** shows the initial temperature distribution of a 1 cm long steel bar at equal distance

Table Q1: The initial temperature distribution of a 1 cm long steel bar

	Point A	Point B	Point C	Point D	Point E
Temperature	A + 90	85	70	55	B + 10

The initial point A is maintained at (A +90) °C, while the last point E is convectively cooled by a coolant at (B + 10) °C for 0.1 seconds. The unsteady state heating equation follows a heat equation, given as

$$\frac{\partial T}{\partial t} - K\frac{\partial^2 T}{\partial x^2} = 0$$

where K is thermal diffusivity of material, x is the longitudinal coordinate of the bar, T is temperature and t is time. The thermal diffusivity of the material is given as $K = 2.5$ cm^2 /s.

Note that A is the first 2 digits of your matrix number, while B is the last two digits of your matrix number. For example, a student with matrix number DD201026 will have the value of $A = 20$ and $B = 26$

(a) Based on the stability requirement, show that the explicit finite difference method is not suitable to be used to find the unknown temperature of each point from 0 second to 0.1 seconds, with $\Delta t = 0.05$ seconds.

(4 marks)

(b) Draw finite-difference grid to predict the temperature at all points up to 0.1 seconds. Use $\Delta t = 0.05$ seconds. Label all the unknown temperature in the grid.

(5 marks)

(c) Determine the unknown temperatures of point A, B, C, D and E at 0.05 seconds.

(11 marks)

Q2 The rate of heat flow (conduction) between two points on a cylinder heated at one end is given by

$$\frac{dQ}{dt} = \lambda A \frac{dT}{dx}$$

where λ is a constant, A is the cylinder's cross-sectional area, Q is heat flow, T is temperature, t is time and r is the distance from the heated end.

Given that

$$\frac{dT}{dx} = \frac{100(L-x)(20-t)}{100-xt}$$

where L is the length of the rod.

If the initial condition is $Q(0) = 0$ and the parameters A is the last digits of your matrix number, $L = 2A$, $x = 2.5$ cm and $\lambda = 0.4$ cal cm/s. For example, student with matrix number DD190010 will have the values of A = 0 and L = 20.

Calculate the heat flow for $t = 0$ to 35 s using Heun's method. Use $h = 5$.

(20 marks)

Q3 Given matrix $A = \begin{pmatrix} 4 & 1 & a \\ 1 & 2 & 1 \\ 0 & 1 & 4 \end{pmatrix}$. Note that $a = b/3$, where b is the last digit of your matrix number. For example, a student with matrix number DD201026 will have the value of $a = 2$.

(a) Find the eigenvalues using characteristic equation.

(7 marks)

(b) Find the smallest eigenvalue and the corresponding eigenvector using Inverse Power Method. Given the initial eigenvector $v^{(0)} = (1 \quad 0 \quad 1)^T$. Iterate from $k = 0$ to $k - 5$.

(10 marks)

(c) Compare the obtained smallest eigenvalue to the answer from characteristic equation in terms of absolute error.

(3 marks)

Q4 (a) Approximate the value of $\frac{1}{\pi}\int_0^{\pi} \cos(0.6A \sin x)\,dx$, using two and three points Gauss Quadrature where A is the last two digits of your matrix number. For example, student with matrix number **AD190300** will have the values of A = 00

(10 marks)

(b) Given a function $f(x) = e^{\sin x} \ln\sqrt{X} + \mathbf{B}$, where B is the last two digits of your matrix number. Determine $f'(0.8)$ by using 2-point forward difference, 2-point backward difference and 3-point Central Difference. For example, student with matrix number **AD190314** will have the values of B = 14

(10 marks)

PART B: ANSWER ONE (1) QUESTION ONLY

Q5 A mass m is released a distance h above a nonlinear spring, as shown in **Figure Q5**. The equation of conservation of energy is given as:

$$0 = \frac{2k_2 d^{5/2}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh$$

Given the parameter values: $k_1 = 1000A$ g/s^2, $k_2 = A$ g/s^{2}m$^{0.5}$, $m = B$ g, $g = 9.81$ m/s^2 and $h = 0.55$ m. Note that A is the first two digits of your matrix number and B is the last two digits of your matrix number. For example, a student with matrix number DD201026 will have the values of $k_1 = 20{,}000$, $k_2 = 20$ and $m = 26$.

(a) Determine the value of d using Newton Raphson method. Use $d_0 = 1.5$. Iterate from $i = 0$ to $i = 4$.

(11 marks)

(b) Determine the value of d using Secant method. Start with interval [1 2]. Iterate from $i = 0$ to $i = 5$.

(9 marks)

Q6 (a) Given the following system of linear equations:

System 1: $$\begin{pmatrix} A & 2 & 3 & 0 \\ 9 & B & 0 & 1 \\ 1 & C & 0 & 9 \\ 0 & 6 & 1 & 8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ -7 \\ 4 \end{pmatrix}$$

System 2: $$\begin{pmatrix} A & B & 0 & 0 \\ 3 & -9 & 7 & 0 \\ 0 & C & -2 & 8 \\ 0 & 0 & 3 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \\ 5 \\ 9 \end{pmatrix}$$

System 3: $$\begin{pmatrix} A & B & 1 \\ C & 0 & -17 \\ 0 & 8 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 19 \\ 7 \\ 0 \end{pmatrix}$$

Note that:

A – First digit of your matrix number
B – Second last digit of your matrix number
C – Last digit of your matrix number

For example, a student with matrix number CD190025 will have the values of A=1, B=2 and C=5

(i) Differentiate the system of linear equations into two different categories.

(2 marks)

(ii) From System 1 to System 3, select the system of linear equations that can be solved by Thomas Algorithm and solve it subsequently.

(8 marks)

(b) For a function f, the divided-differences are given as in **Table Q6(b)**

Table Q6(b): Divided-differences for function f

$x_0 = 0.0$	$f(x_0) = ?$	$f_0^{[1]} = ?$	$f_0^{[2]} = 10 + C$
$x_1 = 0.4$	$f(x_1) = ?$	$f_1^{[1]} = 16 + B$	
$x_2 = 0.7$	$f(x_2) = 9 + A$		

Note that:
A – Last digit of your matrix number
B – Second last digit of your matrix number
C – Last two digits of your matrix number

For example, a student with matrix number DD190025 will have the values of A=5, B=2 and C=25

(i) Determine the missing entries in Table Q6(b).

(6 marks)

(ii) Determine the value of $f(0.25)$.

(2 marks)

(iii) Your friend, Aaron claims that the above data can be used to estimate $f(0.85)$. Do you agree with him? Justify your answer.

(2 marks)

-END OF QUESTIONS-

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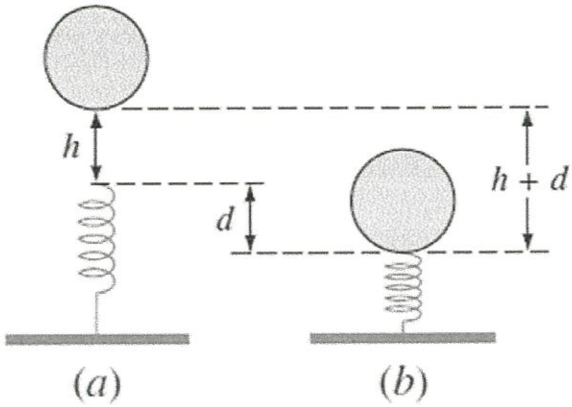

Figure Q5