Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	CALCULUS
COURSE CODE	:	BEE 10103
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION		1.ANSWER ALL QUESTIONS
		2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**
		3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Determine the gradient of the tangent at a point $(1\ ,\ 4)$ to the graph of $xy^2 + 5x = 3x^3y - 2$

(5 marks)

(b) Find derivatives for the following functions.

(i) $y = 3\sin 2t, x = \sqrt{t^2 - 4t}$

(4 marks)

(ii) $y = \frac{5x}{\sinh(6x^2)}$

(4 marks)

(iii) $y = \ln(4t - 2)\cos(1 + e^{-3t})$

(4 marks)

(iv) $f(x) = \ln\frac{\tan 6x}{2x^2 - 1}$

(4 marks)

(c) Solve limit expression of $\lim_{x\to 0}\frac{4\sin^2 x}{x^2}$ by using L'Hopitals's rule.

(4 marks)

Q2 (a) Evaluate the following functions using integration by u-substitution.

(i) $\int (5x^4 - 1)\, e^{(x^5 - x)} dx$

(2 marks)

(ii) $\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\ dx$

(5 marks)

(b) Evaluate the following functions using integration by parts.

(i) $\int x^4 \ln x\, dx$

(5 marks)

(ii) $\int e^x \cos x\, dx$

(6 marks)

(c) Find $\int x^3 \cos(x^2)\, dx$ by combination of substitution and integral by parts methods.

(7 marks)

Q3 (a) Evaluate the following integrals using the tabular method.

(i)

$$\int e^{3x} \cos 3x \, dx$$

(5 marks)

(ii)

$$\int_1^3 x^3 (x-3)^{\frac{5}{2}} \, dx$$

(5 marks)

(b) Compute $\int \frac{x^2+1}{(x+2)^2} \, dx$ using partial fraction method.

(6 marks)

(c) Solve the following integral functions.

(i)

$$\int \frac{\sin^3 \theta}{\cos^2 \theta} \, d\theta$$

(4 marks)

(ii)

$$\int (\sin x)^{\frac{1}{2}} \cos^3 x \, dx$$

(5 marks)

Q4 (a) Differentiate the following expression with respect to x.

$$\tanh^{-1}\left(\frac{1+4x}{1-4x}\right)$$

(4 marks)

(b) If $y = x \sin^{-1} x + \sqrt{1-x^2}$, prove that $\frac{dy}{dx} = \sin^{-1} x$

(4 marks)

(c) Differentiate the following expression with respect to x.

$$x^3 + x \cosh^{-1} 2y = e^y$$

(6 marks)

(d) Evaluate the following integrals.

(i)

$$\int \frac{dx}{\sqrt{3+2x-x^2}}$$

(6 marks)

(ii)

$$\int_0^2 \frac{dx}{25 - 4x^2}$$

(5 marks)

-END OF QUESTIONS-

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FORMULAE

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\ dx = \sinh x + C$$

$$\int \sinh x\ dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\ dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\ dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\ dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\ dx = -\operatorname{csch} x + C$$

Integration Of Inverse Functions

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{|x|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{|x|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{|x|\sqrt{a^2 - x^2}}\, dx = \frac{1}{a}\operatorname{sech}^{-1}\left|\frac{x}{a}\right| + C$$

$$\int \frac{-1}{|x|\sqrt{a^2 + x^2}}\, dx = \frac{1}{a}\operatorname{csch}^{-1}\left|\frac{x}{a}\right| + C$$

$$\int \frac{1}{a^2 - x^2}\, dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & |x| < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & |x| > a \end{cases}$$

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~~FORMULAE~~

TRIGONOMETRIC/ HYPERBOLIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\tan x = \frac{\sin x}{\cos x}$ $\quad \tan x = \frac{1}{\cot x}$	$\tanh x = \frac{\sinh x}{\cosh x}$ $\quad \tanh x = \frac{1}{\coth x}$
$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$	$\operatorname{csch} x = \frac{1}{\sinh x}$ $\quad \operatorname{sech} x = \frac{1}{\cosh x}$
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x\cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x\cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$	$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$
$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$
$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$	$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$
$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$	