UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME   :   CALCULUS I

COURSE CODE   :   BWA 10203

PROGRAMME CODE   :   BWA / BWQ

EXAMINATION DATE   :   JANUARY / FEBRUARY 2022

DURATION   :   3 HOURS

INSTRUCTION   :   1. ANSWER **ALL** QUESTIONS

                                   2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**

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THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

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Q1 (a) Evaluate the limit $\lim_{x \to \infty} (1 + 2x)^{\frac{1}{2 \ln x}}$.

(7 marks)

(b) Determine the value of d so that
$$f(x) = \begin{cases} 4x^2 - 1, & x < 5, \\ 3dx & x \geq 5. \end{cases}$$
is continuous for any value of x.

(4 marks)

(c) By using L'Hopital's rule, find the limits of the following expressions.

(i) $\lim_{x \to 0} \dfrac{e^x - 1}{x^3}$,

(3 marks)

(ii) $\lim_{x \to 0} \dfrac{\cos x + 2x - 1}{3x}$,

(3 marks)

(iii) $\lim_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$.

(3 marks)

Q2 (a) Find $f'(x)$ if $f(x) = e^{\ln \sin 2x} + \ln(x^2 + x - 1) - \cos(3\pi x)$.

(3 marks)

(b) A curve is given by a parametric equation $x = t - \cos t$ and $y = 2 \sin t$. By using parametric differentiation, find $\dfrac{dy}{dx}$ and $\dfrac{d^2 y}{dx^2}$.

(6 marks)

(c) If $\sinh x = \dfrac{e^x - e^{-x}}{2}$ and $\cosh x = \dfrac{e^x + e^{-x}}{2}$, show that $\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$.

(5 marks)

(d) Given a function $f(x) = x^3 - \dfrac{3}{2}x^2 - 6x + 2$, determine the local maximum, local minimum and inflection point (if any) of the function.

(6 marks)

Q3 Evaluate the following integrals.

(a) $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{3 \sin x \cos x}{\sqrt{1 + 3 \sin^2 x}}\, dx$.

(6 marks)

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(b) $\int \sec^5 x \tan x \, dx$

(5 marks)

(c) $\int \dfrac{1}{\sqrt{2-8x-4x^2}} \, dx$

(9 marks)

Q4 (a) If $I_n = \int x^n \sin x \, dx$, prove that $I_n = nx^{n-1} \sin x - x^n \cos x - n(n-1)I_{n-2}$. Hence, evaluate I_3.

(7 marks)

(b) Show that $\dfrac{d}{dx}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. Hence evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x \, dx$.

(6 marks)

(c) Find the arc length of the parametric curve $x = \cos^3 t$ and $y = \sin^3 t$ over the interval $\pi \le t \le \dfrac{3}{2}\pi.$

(7 marks)

Q5 (a) Let R be the region in the first quadrant bounded by the graphs of $x = y^3$ and $x = 4y$ Which is greater, the volume of the solid generated when R is revolved about the x−axis or y−axis?

(8 marks)

(b) If $y = \sqrt{1-x^2} \sin^{-1}(x)$, prove that $(1-x^2)\dfrac{dy}{dx} + xy = 1 - x^2$.

(6 marks)

(c) Evaluate the following integrals.

(i) $\int \dfrac{dx}{1+16x^2}$

(3 marks)

(ii) $\int \dfrac{\cos x}{\sqrt{1+\sin^2 x}} dx$

(3 marks)

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− **END OF QUESTIONS** −