Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	RISK THEORY
COURSE CODE	:	BWA 40803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS A **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**.

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) The owner of an automobile insures its automobile against damage by purchasing an insurance policy with a deductible of RM50. In the event that the automobile is damaged, repair costs can be modelled by a uniform random variable on the cost interval $(0, 250)$. Calculate the standard deviation of the insurance payment in the event that the automobile is damaged. Explain your results.

(17 marks)

(b) An individual with initial wealth of RM400 has a 20% chance of getting in an accident. If he gets in an accident, he will lose RM300, leaving him with RM100; if he does not, he loses nothing. He maximises expected utility and his Von Neumann-Morgenstern utility function is $u(w) = \sqrt{w}$.

(i) What is the certainty equivalent of his wealth?

(6 marks)

(ii) What is the risk premium?

(2 marks)

Q2 (a) A general insurance company has a portfolio of fire insurance policies, which offer cover for just one fire each year. Within the portfolio, there are three types of buildings for which the average cost of a claim and probability of a claim are given in **Table Q2(a)**:

Table Q2(a): Portfolio of Fire Insurance Policies

Type of building	Number of Risks Covered	Average Cost of a Claim	Probability of a Claim
Small	147	12.4	0.031
Medium	218	27.8	0.028
Large	21	130.3	0.017

Assume that the cost of a claim has an exponential distribution, and that all the buildings in the portfolio represent independent risks for this insurance cover.

(i) Calculate the mean and standard deviation of annual aggregate claims from this portfolio of insurance policies.

(12 marks)

(ii) Using a normal distribution to approximate the distribution of annual aggregate claims, calculate the premium loading factor necessary such that the probability that annual aggregate claims exceed premium income is 0.05. Explain your results.

(4 marks)

(iii) Market conditions dictate that the insurer can only charge a premium which includes a loading of 25%. Use a normal approximation, calculate the amount of capital that the insurer must allocate to this line of business in order to ensure that the probability that annual aggregate claims exceed premium income and capital is 0.05. Justify the results.

(4 marks)

(b) Consider a portfolio of 32 policies. For each policy, the probability q of a claim 1/6 and B, the benefit amount given that there is a claim, has probability distribution function:

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Let S be the total claims for the portfolio. Using a normal approximation, estimate the probability of aggregate claim which exceeds the claim amount 4, i.e. $P(S > 4)$. Explain your results.

(5 marks)

Q3 (a) A portfolio consists of two types of policies. For type I, the number of claims in a year has a Poisson distribution with mean 1.5 and the claim sizes are exponentially distributed with mean 5. For type II, the number of claims in a year has a Poisson distribution with mean 2 and the claim sizes are exponentially distributed with mean 4. Let S be the total amount claimed on the whole portfolio in one year. All polices are assumed to be independent.

(i) Determine the mean and variance of S.

(5 marks)

(ii) Derive the moment generating function of S and show that S has a compound Poisson distribution.

(8 marks)

(b) Claim frequency follows a Poisson process with a rate of 10 per year. Claim severity is exponentially distributed with mean 2000. The method of moments is used to estimate the parameters of a lognormal distribution for the aggregate losses. Using the lognormal approximation, calculate the probability that annual aggregate losses exceed 105% of the expected annual losses. Explain your results.

(12 marks)

Q4 (a) Claims occur in a Poisson process rate 20. Individual claims are independent random variables with density

$$f(x) = \frac{3}{(1+x)^4}, \quad x > 0,$$

independent of the arrivals process.

(i) Calculate the mean and variance of the total amount claimed at time $t = 2$.

(2 marks)

(ii) Using a normal approximation, derive approximately the probability of ruin at $t = 2$ if the premium loading factor is 30% and the initial surplus is $u = 10$.

(3 marks)

(b) An insurer knows from past experience that the number of claims received per month has a Poisson distribution with mean 15, and that claim amounts have an exponential distribution with mean 500. The insurer uses a security loading of 30%. Calculate the insurer's adjustment coefficient and give an upper bound for the insurer's probability of ruin, if the insurer sets aside an initial surplus of 1,000.

(8 marks)

(c) Write down the equation for the adjustment coefficient for personal accident claims if 90% of claims are for RM10,000 and 10% of claims are for RM25,000, assuming a proportional security loading of 20%. Show that this equation has a solution in the range $0.00002599 < R < 0.00002601$.

(12 marks)

- END OF QUESTIONS -