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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	ACTUARIAL MATHEMATICS I
COURSE CODE	:	BWA 31403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**.

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Let $F_0(t)=1-e^{-\lambda t}$, where $\lambda>0$.

(i) Show that $S_x=e^{-\lambda t}$.

(5 marks)

(ii) Show that $\mu_x=\lambda$.

(5 marks)

(b) Mortality for a population consisting of females and males follow a select and ultimate table, an extract of which is given in **Table Q1(b)**. Males have a 3-year select period while females have a 2-year select period. Assume mortality follows the Uniform Distribution of Death (UDD) between integral ages.

Table Q1(b)

Males							Females				
x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	l_{x+3}	$x+3$		x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
50	80960	79827	78522	77025	53		50	70764	69124	67224	52
51	79530	78334	76958	75382	54		51	68823	67118	65146	53
52	78021	76760	75312	73655	55		52	66805	65036	62993	54
53	76430	75103	73581	71842	56		53	64711	62879	60768	55
54	74756	73362	71765	69944	57		54	62544	60651	58475	56
55	72998	71535	69863	67958	58		55	60305	58354	56117	57

(i) Propose the probability that a randomly chosen female from this population 51.25, with select age 50, will die within the next 3 years and 9 months.

(7 marks)

(ii) Propose the probability that a randomly chosen male from this population, at select age 51, will survive within the ages of 52.35 and 56.75.

(8 marks)

Q2 (a) Consider the following two present value random variables:

$$Z_1=\begin{cases}10000v^{T_x}, & T_x\le 15\\ 20000v^{15}, & T_x>15.\end{cases}$$

$$Z_2=\begin{cases}0, & T_x\ge 5\\ 10000v^{T_x}, & 5<T_x\le 15\\ 20000v^{15}, & T_x>15.\end{cases}$$

(i) Describe the insurance policies represented by Z_1 and Z_2 from the perspective of benefit payments.

(4 marks)

(ii) Express the expected present value of Z_1, $E(Z_1)$ and the expected present value of Z_2, $E(Z_2)$ using actuarial symbols.

(5 marks)

(iii) Table **Q2(a)(iii)** shows the actuarial present values $\bar{A}_x$ and the expected number of survival l_x for the insurance policies.

Table Q2(a)(iii)

x	$\bar{A}_x$	l_x
x	0.166117	93132
$x+5$	0.207180	91641
$x+15$	0.314208	86409

Suppose the effective rate of interest, $i = 6\%$ per year, calculate $E(Z_1)$ and $E(Z_2)$.

(5 marks)

(b) A 40-year-old man buys a whole life insurance policy from Great Eastern Malaysia Berhad. The policy will pay RM150,000 at the end of the year of his death. You are given:

- $\mu = 0.01$
- $\delta = 0.07$
- $i = 10\%$

(i) Determine the type of this insurance based on the time of benefit payment.

(2 marks)

(ii) Determine the Actuarial Present Value (APV) of this insurance.

(3 marks)

(iii) Calculate the standard deviation of the present value of this insurance.

(6 marks)

Q3 (a) Jamilah, currently age 40, just joined XYZ company with a starting salary of RM 6,250 per month. XYZ provides a benefit, payable at the moment of her death, equal to 4 times her total annual salary if she dies while employed with the company and before reaching retirement age 65. Assume that Jamilah intends to work for the company until retirement age 65. You are given:

- Mortality follows de Moivre's law with $\omega = 100$.
- $\delta = 7\%$.

Suppose that Jamilah's salary increases continuously at an annual rate of 4%, that is, his salary at time, t from start of employment is $75,000e^{0.04t}$, for $t \geq 0$.

(i) Express the present value random variable of Jamilah's death benefits.
(3 marks)

(ii) Calculate the actuarial present value of her death benefits.
(4 marks)

(iii) Calculate the variance of the present value of her death benefits.
(5 marks)

(b) Assuming you are currently working in the product development department of an insurance company. You are proposing to your head of department to improve a life annuity product by adding a death benefit that will be paid at the end of the year of death. This new payment is in addition to an existing payment of RM12,000 to the annuitant at the beginning of each year. Assuming a discount rate, d, of 8%, determine the death benefit that maximizes the variance of the present value random variable of the new product.
(5 marks)

(c) If $A_x = 0.25, A_{x+20} = 0.40$ and $A_{x:\overline{20|}} = 0.55$, calculate

(i) $A_{x:\overline{20|}}^{\;\;\;\;1}$, and
(4 marks)

(i) $A^{1}_{x:\overline{20|}}$.
(4 marks)

Q4 (a) The net premiums are determined by two factors which are beyond the control of an insurance company. Identify these two factors.
(4 marks)

(b) For a 10-year term life insurance policy issued to a policyholder aged 50, you are given:

- The death benefit of RM 100 is payable at the end of the year of death,
- A level premium is paid at the beginning of each year during the term of the policy,
- Mortality follows the Illustrative Life Table in **Table Q4(b),**
- $i = 0.06$,
- Net premium is calculated according to the actuarial equivalence principle.

Using Illustrative Life Table as shown in **Table Q4(b)**, calculate the net premium reserve at the end of year 5.

Table Q4(b)

Age	$\ddot{a}_x$	$1000A_x$	$1000\left({}^2A_x\right)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$
50	13.2668	249.05	94.76	721.37	510.81
51	13.0803	259.61	101.15	719.17	506.78
52	12.8879	270.50	107.92	716.76	502.40
53	12.6896	281.72	115.09	714.12	497.64
54	12.4856	293.27	122.67	711.24	492.47
55	12.2758	305.14	130.67	708.10	486.86
56	12.0604	317.33	139.11	704.67	480.79
57	11.8395	329.84	147.99	700.93	474.22
58	11.6133	342.65	157.33	696.85	467.12
59	11.3818	355.75	167.13	692.41	459.46
60	11.1454	369.13	177.41	687.56	451.20

(9 marks)

(c) Suppose that mortality follow **Table Q4(c)**. Calculate the level annual benefit premium payable in semiannual installments for a 10,000, 20-year endowment insurance with proceed paid at the end of the policy year of death (discrete) issued to (50). Assume that $i = 0.06$ and that uniform distribution of death assumption holds over each year of age.

Table Q4(c)

x	$1000q_x$	$1000A_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$
40	2.78	161.32	536.67	274.14
41	2.98	168.69	534.99	271.12
42	3.20	176.36	533.14	267.85
43	3.44	184.33	531.12	264.31
44	3.71	192.61	528.92	260.48
45	4.00	201.20	526.52	256.34
46	4.31	210.12	523.89	251.88
47	4.66	219.36	521.03	247.08
48	5.04	228.92	517.91	241.93
49	5.46	238.82	514.51	236.39
50	5.92	249.05	510.81	230.47

(12 marks)

–END OF QUESTIONS–