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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	NUMERICAL METHOD
COURSE CODE	:	BWA 21503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**.

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 (a) Compare Newton-Raphson method and fixed-point iteration method by giving **ONE (1)** advantage and **ONE (1)** disadvantage of both methods.

(4 marks)

(b) The velocity v of a falling parachutist is given by

$$v = \frac{gm}{c}\left(1 - e^{-(c/m)t}\right),$$

where

$g = 9.8\,\text{ms}^{-2}$ is the gravitational force, and

$c = 15\ \text{kgs}^{-1}$ is the drag coefficient.

Taking initial guess of the parachutist mass m as between 60 kg to 70 kg, calculate the mass m using false-position method when the velocity of the falling parachutist is $35\,\text{ms}^{-1}$ at time $t = 9\,\text{s}$. Do the iteration until $|f(m_i)| < \varepsilon = 0.0005$.

(8 marks)

(c) A steady-state concentration of a substance that reacts with first-order kinetics in an axially-dispersed plug-flow reactor can be expressed as

$$D\left(\frac{c_{i-1} - 2c_i + c_{i+1}}{(\Delta x)^2}\right) - U\left(\frac{c_{i+1} - c_{i-1}}{2(\Delta x)}\right) - kc_i = 0,$$

where D is dispersion coefficient (m^2/hr), c_i is concentration at node i (mol/L), x is distance (m), U is fluid velocity (m/hr) and k is reaction rate per hour. The parameter values for the mass balance problem are $D = 2$, $U = 1$, $k = 0.2$, $\Delta x = 2.5$, $c(x=0) = 80$ and $c(x=10) = 20$.

(i) Restructure the above problem into a system of linear equations.

(7 marks)

(ii) Hence, solve the system by using Gauss-Seidel iteration method with initial concentrations, $c_1 = 54$, $c_2 = 37$ and $c_3 = 26$. Do the iteration until $\max\limits_{1 \le i \le n} \{|c_i^{(k+1)} - c_i^{(k)}|\} < \varepsilon = 0.005$.

(6 marks)

Q2 (a) Consider the data points (0, 1), (1, 3), (1.5, 0) and (2, α). Use Lagrange polynomial interpolation to find the value of α if the coefficient of x^3 in the polynomial is 5.

(9 marks)

(b) Given $f(x) = e^{2x} - x^3 + 3x - 1$.

(i) Approximate values for $f'(2)$ using 3 point-difference formulas if $h = 0.1$.

(7 marks)

(ii) Based on **Q2(b)(i)**, prove that 3 point-central gives the best approximation.

(3 marks)

(c) A racehorse is running a 1 km race. A timer is set to record the distance, d (in meters) of the racehorse for every 10 seconds, t as shown in the **Table Q2(c)**.

Table Q2(c): Distance, d (in meters) of the racehorse for every 10 seconds, t

t	0	10	20	30	40	50	60	70	80	90	100
d	0	45	104	196	310	408	495	620	721	816	902

(i) During which part of the race is the horse running the fastest?

(2 marks)

(ii) Calculate the acceleration of the horse at its fastest part.

(4 marks)

Q3 (a) The capacity of a battery is measured by $\int I\, dt$ where I is the current. Estimate the capacity of the battery based on **Table Q3(a)** using the combination of Simpson's rule. Explain the requirement in solving this problem using Simpson's rule. Give the answer in four (4) digit rounding arithmetic.

Table Q3(a): Current of a battery

Time (hour)	Current, I (A)
0	25.20
5	29.00
10	31.80
15	34.02
20	36.50
25	33.71
30	31.24
35	29.67
40	28.53
45	29.44
50	31.76
55	32.70

(12 marks)

(b) Consider a matrix

$$A = \begin{pmatrix} 4 & -1 & 1 \\ 1 & 1 & 1 \\ -2 & 0 & -6 \end{pmatrix}.$$

(i) Determine the interval of which the eigenvalues of matrix A above are contained by using Gerschgorin's theorem.

(7 marks)

(ii) Find the dominant eigenvalue and corresponding eigenvector for matrix A by using power method with $v^{(0)} = (1 \quad 0 \quad 0)^T$. Do your calculation until 6 iterations.

(6 marks)

Q4 (a) The following methods can be used to solve an ODE $x' = f(t,x)$.

(i) $x_{n+1} = x_n + \frac{1}{2}[f(t_n, x_n) + f(t_{n+1}, x_n + f(t_n, x_n)h)]h$.

(ii) $x_{n+1} = x_n + \frac{1}{2}[3f(t_n, x_n) - f(t_{n-1}, x_{n-1})]h$.

(iii) $x_{n+1} = x_n + \frac{1}{2}[f(t_n, x_n) + f(t_{n+1}, x_{n+1})]h$.

For each method, analyse if it has the following properties: single-step method, implicit method and Runge-Kutta type method.

(6 marks)

(b) Given $y' = -2xy^2$, $y(0) = 1$, with $h = 0.2$ on the interval $[0, 0.4]$. Solve the initial value problem by using

(i) Huen's method, and

(4 marks)

(ii) Fourth-order Runge-Kutta (RK4) method.

(6 marks)

(iii) Hence, compare the effectiveness of both methods in **Q4(b)(i)** and **Q4(b)(ii)** if the exact solution is $y(x) = \frac{1}{1+x^2}$.

(3 marks)

(c) Provide **THREE (3)** differences on fourth-order Runge-Kutta (RK4) method over Huen's method in solving first order initial value problems.

(6 marks)

- END OF QUESTIONS -