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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	ENGINEERING MATHEMATICS II
COURSE CODE	:	DAE 23403
PROGRAMME CODE	:	DAE
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWERS **ALL** QUESTIONS IN SECTION A AND ANSWER **THREE (3)** QUESTIONS IN SECTION B. 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK.**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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SECTION A

Q1 Solve the following equations by using the method of undetermined coefficient.

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 2\cos 2x$.

(8 marks)

(b) $\frac{d^2y}{dx^2} - 4y = x + 5\sin 3x$; $y(0) = 2$ and $y'(0) = 0$.

(12 marks)

Q2 By using the method of variation of parameter to find the general solution of the given second order differential equation.

(a) $y'' - 6y' + 9y = (3 - x)e^{3x}$.

(10 marks)

(b) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = -\frac{x}{\pi} - 3$

(10 marks)

SECTION B

Q3 (a) Find the integrals of :

(i) $\int (8x^3 + 3e^{-5x})\, dx$.

(4 marks)

(ii) $\int \left(\cos 7x - \frac{4}{(3x+1)} \right) dx$.

(4 marks)

(iii) $\int x^2 \ln x \, dx$.

(5 marks)

(b) Find the approximate value for $\int_0^1 \sqrt{x^2 + 1}\, dx$ using trapezoidal rule by taking step size, $h = 0.2$.

(7 marks)

Q4 (a) Given two curves $y=\sqrt{x-1}$ and $y=(x-1)^2$.

(i) Sketch the graphs of the curves.

(5 marks)

(ii) By using the cylindrical shells method, find the volume of the solid generated when the bounded region is rotated about $x-$axis.

(8 marks)

(b) Find the arc length of the curve $y=\frac{4}{3}x^{\frac{3}{2}}-1$ when $1\le x\le 2$.

(7 marks)

Q5 (a) Find the solution of the given linear differential equation.

$$(x+2)\frac{dy}{dx}+y=(x+2)e^{2x}, \qquad \text{IVP: } y(0)=2.$$

(10 marks)

(b) Given the first order differential equation :

$$\left(y-xy^2+2ye^x\right)dx+\left(x-x^2y+2e^x\right)dy=0.$$

(i) Show that the equation is an exact ordinary differential equation.

(2 marks)

(ii) Find the general solution of the equation.

(8 marks)

Q6 (a) As a worker in a factory, you need to remove a heavy metal with its core temperature of 1000° F from a furnace and placed the metal on a table in a room that had a constant temperature of 72° F. One hour after it is removed the core temperature is 910° F. The temperature of the metal must be below 540° F before you can transfer it to the next section.

(i) Given $\frac{dT}{T-T_s}=-kdt$. Show that $T-T_s=Ae^{-kt}$.

(4 marks)

(ii) By using $T-T_s=Ae^{-kt}$, with $T(0)=1000^\circ$ F and $T_s=72^\circ$ F, find the constant A. Hence find $T(t)$.

(4 marks)

(iii) Given the observed temperatures of the metal, given $T(1)=910^{\circ}\text{F}$, find the constant k.

(4 marks)

(iv) Find the time taken for the temperature of the metal to be below 540°F.

(3 marks)

(b) The world population growth is described by $y(t)=y_0e^{k(t-t_0)}$ with t measured in years.

(i) If the population increased 2011 by 3% from 2010 to 2011, find k.

(3 marks)

(ii) If the population in $t_0=2010$ was 5 million people, find the actual population for 2020 predicted by the given equation.

(2 marks)

Q7 (a) By using the integration by parts to show that $\int_0^2 \frac{1}{\sqrt{x}}e^{\frac{1}{2}x}dx = 2\sqrt{x}e^{\frac{1}{2}x} - \int_0^2 \sqrt{x}e^{\frac{1}{2}x}dx$.

(3 marks)

(b) By using Simpson's Rule with $h = 0.4$, find the approximate value of $\int_1^3 \frac{3x}{x+5}dx$.

(5 marks)

(c) The equations of two curves are given by $y=x^2-1$ and $y=\frac{6}{x^2}$.

(i) Sketch the two curves on the same coordinates axes.

(3 marks)

(ii) Find the coordinates of the points of intersection of the two curves.

(3 marks)

(iii) Calculate the volume of the solid formed when the region bounded by the two curves and the line $x = 1$ is revolved completely about the $y-$axis.

(6 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

SEMESTER / SESSION : SEM I 2021/2022
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Formula

Table 1: Characteristic Equation and General Solution

Homogeneous Differential Equation: $ay'' + by' + cy = 0$

Characteristics Equation: $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case	Roots of Characteristics Equation	General Solution
1	Real and Distinct: $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	Real and Equal: $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	Complex Roots: $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2: Particular Solution of Nonhomogeneous Equation

$ay'' + by' + cy = f(x)$

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)$
$Ce^{\alpha x}$	$x^r\left(Pe^{\alpha x}\right)$
$C\cos\beta x$ or $C\sin\beta x$	$x^r\left(P\cos\beta x + Q\sin\beta x\right)$
$P_n(x)e^{\alpha x}$	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$ or	$x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\cos\beta x$ + $x^r\left(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0\right)\sin\beta x$

Notes: r is the smallest non negative integer to ensure no alike term between $y_p(x)$ and $y_h(x)$.

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Table3: Variation of Parameters Method

Homogeneous solution, $y_h(x) = Ay_1 + By_2$	
Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$	
$u_1 = -\int \frac{y_2 f(x)}{aW} dx + A$	$u_2 = \int \frac{y_1 f(x)}{aW} dx + B$
General solution, $y(x) = u_1y_1 + u_2y_2$	

Table 4: Trigonometry Identities

$\sin^2 t + \cos^2 t = 1$
$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$
$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$

Table 5: Partial Fraction

$\frac{a}{(s+b)(s-c)} = \frac{A}{s+b} + \frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)} = \frac{A}{s} + \frac{B}{s-b} + \frac{C}{s-c}$
$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)} = \frac{A}{(s+b)} + \frac{Bs+C}{(s^2+c)}$

FINAL EXAMINATION

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Table 6: Integration and Differentiation

Integration	Differentiation
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{dx} x^n = nx^{n-1}$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\frac{d}{dx} \ln x = \frac{1}{x}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b} \ln\lvert a-bx\rvert + C$	$\frac{d}{dx} \ln(ax+b) = \frac{a}{ax+b}$
$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$	$\frac{d}{dx} e^{ax} = ae^{ax} n$
$\int \sin ax\, dx = -\frac{1}{a} \cos ax + C$	$\frac{d}{dx} \sin ax = a \cos ax$
$\int \cos ax\, dx = \frac{1}{a} \sin ax + C$	$\frac{d}{dx} \cos ax = -a \sin ax$
$\int \sec^2 x\, dx = \tan x + C$	$\frac{d}{dx} \tan x = \sec^2 x$
$\int \csc^2 x\, dx = -\cot x + C$	$\frac{d}{dx} \cot x = -\csc^2 x$
$\int u\, dv = uv - \int v du$	$\frac{d}{ds}(uv) = u\frac{dv}{ds} + v\frac{du}{ds}$
$\int_a^b f(x) dx = F(b) - F(a)$	$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$

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Area of Region

$$A = \int_a^b [f(x) - g(x)]\,dx \quad \text{or} \quad A = \int_c^d [w(y) - v(y)]\,dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\, f(x)\, dx \quad \text{or} \quad V = \int_c^d 2\pi y\, f(y)\, dy$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[\left(f(a) + f(b)\right) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a+ih) + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(a+ih) \right]; \quad n = \frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih) \right]; \quad n = \frac{b-a}{h}$$