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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	DAU 34403
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN PART A AND **THREE (3)** QUESTIONS IN PART B 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSED BOOK**

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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PART A

Q1 (a) Find the inverse of the following Laplace expressions:

(i) $\frac{6s+1}{s^2+4}$.

(4 marks)

(ii) $\frac{7}{s^3}-\frac{5}{s}+\frac{1}{s+3}$.

(4 marks)

(iii) $\frac{8}{3s^2+12}-\frac{4}{s^2-36}$.

(5 marks)

(b) (i) Express $\frac{s-3}{s^2-5s-14}$ as partial fractions.

(5 marks)

(ii) Determine the inverse Laplace of the partial fraction from **Q1(b)(i).**

(2 marks)

Q2 (a) By using Laplace Transform, find the solution $y(t)$ for each of the following differential equations:

(i) $y'+4y=3e^{-4t}$, $y(0)=1$.

(8 marks)

(ii) $y''-7y'+10y=3$, $y(0)=1$, $y'(0)=5$.

(12 marks)

PART B

Q3 (a) Given a first order differential equation:

$$(xy)dy-(y^2-x^2)dx=0.$$

(i) Show that the differential equation above is a homogenous equation.

(2 marks)

(ii) Solve the homogenous equation with condition $y(1)=2$.

(7 marks)

(b) Given $\left(3x^2+\frac{7}{3}xy^3\right)dx+\left(\frac{7}{2}x^2y^2-2y^2\right)dy=0.$

(i) Show that the differential equation above is an exact equation.

(3 marks)

(ii) Then, solve the equation from **Q3(b)(i).**

(8 marks)

Q4 (a) The rate of cooling of a body is given by equation

$$\frac{dT}{dt}=-k(T-10),$$

where T is the temperature in degree Celcius, k is a constant and t is the time in minutes. When $t=0, T=90°C$ and when $t=5, T=60°C$. Show that when $t=20,\ T=22.21°C$.

(10 marks)

(b) A certain city had a population of 25000 in year 1990 and a population of 30000 in year 2000. Assume that its population will continue to grow exponentially at a constant rate.

(i) Determine at which year the population would reach 1.2 million.

(8 marks)

(ii) Calculate the population of that city in the year 2030.

(2 marks)

Q5 (a) Solve the second order homogeneous differential equation:

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-3y=0 \text{ if given } y(0)=1 \text{ and } y'(0)=2.$$

(8 marks)

(b) Find the particular solution of nonhomogenous differential equation:

$$\frac{d^2y}{dx^2}-12\frac{dy}{dx}+36y=-3e^{6x}.$$

(3 marks)

(c) Using undetermined coefficient method, solve the differential equation:

$$\frac{d^2y}{dx^2}-9y=4\sin 2x.$$

(9 marks)

Q6 Find the general solution of the following second order non-homogeneous differential equation by using the method of variation parameters.

(a) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=4e^{2x}.$

(9 marks)

(b) $\frac{d^2y}{dx^2}+4y=3\cos 2x.$

(11 marks)

Q7 (a) Find the Laplace transform of the following functions:

(i) $f(t)=8-\sin 2t+\cos 3t-3e^{t}+2e^{-t}.$

(6 marks)

(ii) $f(t)=e^{3t}\cos 5t.$

(4 marks)

(iii) $f(t)=5e^{t}-t\sin 3t.$

(5 marks)

(b) Show that $L\{e^{3t}\cosh 2t+t^4e^{-t}\}=\frac{s-3}{s^2-6s}+\frac{24}{(s+1)^5}.$

(5 marks)

–END OF QUESTIONS–

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Table 1: Characteristic Equation and General Solution

Differential equation : $ay'' + by' + cy = 0$ Characteristic equation : $am^2 + bm + c = 0$		
Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2: Method of Undetermined Coefficients

Case	$F(x)$	$y_p(x)$
1	Simple polynomial: $A_0 + A_1x + ... + A_nx^n$	$x^r\left(B_0 + B_1x + ... + B_nx^n\right), r = 0,1,2,...$
2	Exponential function: $Ce^{\alpha x}$	$x^r\left(Ke^{\alpha x}\right), r = 0,1,2,...$
3	Simple trigonometry: $C\cos\beta x$ or $C\sin\beta x$	$x^r\left(P\cos\beta x + Q\sin\beta x\right), r = 0,1,2,...$

Table 3: Method of Variation of Parameters

Homogeneous solution, $y_h(x) = Ay_1 + By_2$

Wronskian function, $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

$u_1 = -\int \frac{y_2 f(x)}{aW} dx$ $\quad$ $u_2 = \int \frac{y_1 f(x)}{aW} dx$

Particular solution, $y_p = u_1y_1 + u_2y_2$

Final solution, $y(x) = y_h(x) + y_p(x)$

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Table 4: Trigonometry Identities

$$\cos^2 x + \sin^2 x = 1$$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$2\sin x \cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x \cos y = \cos(x+y) + \cos(x-y)$$

Table 5: Differentiation and Integration

Differentiation	Integration
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
$\frac{d}{dx}\ln x = \frac{1}{x}$	$\int \frac{1}{x}\, dx = \ln\|x\| + C$
$\frac{d}{dx}\ln\|ax+b\| = \frac{1}{ax+b}$	$\int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln\|ax+b\| + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x dx = e^x + C$
$\frac{d}{dx}\sin ax = a\cos ax$	$\int \cos ax\, dx = \frac{1}{a}\sin ax + C$
$\frac{d}{dx}\cos ax = -a\sin ax$	$\int \sin ax\, dx = -\frac{1}{a}\cos ax + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x\, dx = \tan x + C$
$\frac{d}{dx}\cot x = -\csc^2 x$	$\int \csc^2 x\, dx = -\cot x + C$

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Table 6: Laplace and Inverse Laplace Transforms

$L\{f(t)\}=\int_0^{\infty} f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n=1,2,\ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
The First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t), n=1,2,\ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Initial Value Problem	
$L\{y(t)\}=Y(s)$ $L\{y'(t)\}=sY(s)-y(0)$ $L\{y''(t)\}=s^2Y(s)-sy(0)-y'(0)$	

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Table 7: Formula for Growth and Decay and Newton's Cooling Law

Growth and Decay

$$N = Ae^{-kt}$$

Newton's Cooling Law

$$T = (T_0 - T_s)e^{-kt} + T_s$$

Table 8: Partial Fraction

$$\frac{a}{(s+b)(s+c)} = \frac{A}{s+b} + \frac{B}{s+c}$$

$$\frac{a}{s(s+b)(s+c)} = \frac{A}{s} + \frac{B}{s+b} + \frac{C}{s+c}$$

$$\frac{a}{(s+b)^2} = \frac{A}{s+b} + \frac{B}{(s+b)^2}$$

$$\frac{a}{(s+b)(s^2+c)} = \frac{A}{s+b} + \frac{Bs+C}{s^2+c}$$