CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHODS
COURSE CODE	:	BFC25203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION		1. ANSWER ALL QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) **Table Q1 (a)** shows the settlement of Batu Pahat municipal solid waste landfills over 5-years period.

(i) Develop the natural cubic spline by using data given in the table.

(14 marks)

(ii) Based on **Q1(a)(i)** verify that f_2 (3) and f_3 (3) = 19.

(1 mark)

(b) The experimental result for a cantilever beam's deflection response to the exposure to a 10 kN/m load are shown in **Table Q1 (b)**. Given the slope of the beam is $\theta = d'(x)$ and the bending moment of the beam is $M = \theta'(x)$

(i) By using 3-point central, 3-point forward and 5-point central formulas, approximate the values of the slope of the beam at 3.0 m length. Do all calculation in 4 decimal places.

(3 marks)

(ii) Based on **Q1(b)(i)**, identify the method that could be capable of generating the most accurate approximation and provide a justification to support your answer. Given the exact solution of the slope of beam the is $0.0625\, x^3 - 1.125x^2 + 6.73\, x$.

(5 marks)

(iii) Evaluate **two (2)** approximate values of bending moment of the beam at the deflection of 34 mm. Do all calculation in 4 decimal places.

(2 marks)

Q2 (a) Approximate $\int_0^3 2 + \sin\left(2\sqrt{x}\right) dx$ by using an appropriate Simpson's rule with $n = 9$ and state your reason. Do all calculations in 3 decimal places.

(10 marks)

(b) Given $\int_1^3 \frac{2t^2}{6+6t^4} dt = \int_{-1}^1 \frac{(x+2)^2}{3+3(x+2)^4} dx$

(i) By taking $t = \frac{(b-a)x+(b+a)}{2}$, show that the two integrals above are equivalent.

(5 marks)

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(ii) Then, find the value of $\int_1^3 \frac{2t^2}{6+6t^4}dt$ by using the 2-point and 3-point Gauss Quadrature formula. Find the absolute error for each point if the exact solution is the answer base on calculator. Do all calculations in 3 decimal places.

(10 marks)

Q3 The stability of the bridge construction can be calculated and determined by the natural frequency of a bridge system (smallest magnitude eigenvalue) and its corresponding eigenvector in the matrix form as:

$$C = \begin{bmatrix} 5 & -2 & 1 \\ -2 & 2 & 0 \\ 2 & -2 & 3 \end{bmatrix}$$

(a) Use $v^{(0)} = (1 \quad 0 \quad 1)$ and stop the iteration until $|m_{k+1} - m_k| < 0.005$. Do all calculations in 3 decimal places.

(25 marks)

Q4 (a) Fourth order Runge-Kutta (4th-RK) method:

(i) Solving a first Ordinary Differential Equation (ODE) using the RK4.

$$\frac{dy}{dx} = -1.2y + 7e^{-0.3x}$$

from $x = 0$ to $x = 2.5$ with the initial condition $y = 3$ at $x = 0$. Using $h = 0.5$.

(6 marks)

(ii) Sketch the RK4 results with the exact (analytical) solution:

$$y = \frac{70}{9}e^{-0.3x} - \frac{43}{9}e^{-1.2x}$$

Using $h = 0.05$.

(9 marks)

(b) Consider a steel rod AB of 4 meters long, with taking $\Delta x = h = 1$, is subjected to a temperature of 0°C at the point A (left end) and is maintained at 10°C at the point B (right end) until a steady state of temperature along the bar is achieved. At t= 0s, however the end of point B is suddenly reduced to 0°C while the other points are kept at the same temperature. By taking $k = \Delta t = 0.2$s until $t = 0.4$s only, use the implicit method to solve the heat equation:

$$\frac{\partial u}{\partial t} = \frac{\partial u^2}{\partial x^2}$$

(10 marks)

-END OF QUESTIONS-

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Table Q1(a)

Year	1	2	3	5
Soil settlement (cm)	3	6	19	99

Table Q1(b)

Beam's length, x (m)	Beam's deflection, d (mm)
0.5	0.7979
1.0	3.0156
1.5	6.4072
2.0	10.7500
2.5	15.8447
3.0	21.5156
3.5	27.6100
4.0	34.0000
4.5	40.5791
5.0	47.2656
5.5	54.0001
6.0	60.7500

Formulae

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$; $f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\, x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\, x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3, \ldots, n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3, \ldots, n-1$$

$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,..,n-2,$

When; $m_0 = 0, m_n = 0,$
$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, k = 0,1,2,3,\ldots,n-2$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1}-x)^3 + \frac{m_{k+1}}{6h_k}(x-x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1}-x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x-x_k) \quad , k = 0,1,2,3,\ldots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h)-f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x)-f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x)-4f(x-h)+f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h)-2f(x)+f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\,odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\,even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\left[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3})\right]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method :$v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

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where $k_1 = hf(x_i, y_i)$ $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$
$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat Equation: Crank-Nicolson Implicit Finite-Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i.j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \quad \frac{u_{i+1,j} - 2u_{i.j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j} \quad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2 \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$