Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHODS / ENGINEERING MATHEMATICS IV
COURSE CODE	:	BEE 32402/ BEE 31602
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 The velocity, v of an object at various points in time, t is given in **Table Q1**.

Table Q1

$t, (s)$	0.99	1.02	1.05	1.08	1.11
$v, (m/s)$	1.4072	1.4284	1.45	1.4792	1.4940

(a) Estimate the acceleration, $a = 2\frac{dv}{dt}$, for the object at $t = 1.05s$ by applying all appropriate numerical first order derivative methods to the following questions:

(i) Interval of $0.03\ s$

(10 marks)

(ii) Interval of $0.06\ s$

(10 marks)

(b) Identify the best method in estimating the acceleration of the signals with a concise justification if the exact solution is 1.6546.

(5 marks)

Q2 (a) A basketball player makes a successful shot from the free throw line. Suppose that the path of the ball from the moment of release to the moment it enters the hoop is described by

$$y = 2.15 + 2.09x - 0.41x^2\,, \qquad 0 \leq x \leq 0.24$$

where x is the horizontal distance (in meters) from the point of release, and y is the vertical distance (in meters) above the floor.

(i) Find the distance of the ball travels from the moment of release to the moment it enters the hoop, by using trapezoidal rule and appropriate Simpson's rule with $h = 0.3$.

[Hint: Arc length of the curve, $L = \int_a^b \sqrt{1 + 2\left(\frac{dy}{dx}\right)}\, dx$]

(12 marks)

(ii) Calculate the exact solution of the traveled distance by using scientific calculator

(2 marks)

(iii) Calculate the absolute error for each method from **Q2(a)(i)**.

(2 mark)

(iv) Point out which method approximates better.

(1 mark)

(b) The **Table Q2** below gives the values of distance traveled by car at various time, t from a tollgate at highway. Calculate the distance traveled, x by referring the following data using suitable Simpson's rules.

Table Q2

Time, t (minute)	3	5	7	9	11	13	15
Distance traveled, x (km)	4.600	8.030	11.966	16.885	19.904	21.504	23.134

(8 marks)

Q3 According to Kirchhoff's voltage law, a simple series RL circuit that can consist of a resistor, an inductor and a power supply can be represented by the following equation.

$$L\frac{di}{dt} + Ri = E(t)$$

Given $E(t) = 120V$, $L = 3\text{H}$, $R=15\Omega$, $i = 3.2101A$ when $t = 0.10\ s$

(a) Calculate the $i(t)$ between $0.10\ s$ and $0.15\ s$ with an interval of $0.10\ s$ using Euler's method.

(8 marks)

(b) Given $i = 4.2211A$ when $t = 0.15\ s$, calculate the $i(t)$ between $0.10\ s$ and $0.15\ s$ with an interval of 0.01s using finite-different method.

(10 marks)

(c) Find the absolute errors at each estimation at the **Q3(a)** and **Q3(b)** if the exact solution is $i(t) = 8(1 - e^{-5t})$

(7 marks)

Q4 (a) The temperature distribution $u(x,t)$ of a one-dimensional silver rod is governed by the heat equation as follows.

$$\frac{\partial u}{\partial t} = 0.5\frac{\partial^2 u}{\partial x^2}$$

Given the boundary conditions $u(0,t) = t^2$, $u(0.6,t) = 6t + 0.12$, for $0 \le t \le 0.04s$ and the initial condition $u(x,0) = x(0.8 - x)$ for $0 \le x \le 0.6\text{mm}$, analyze the temperature distribution of the rod with $\Delta x = 0.2\text{mm}$ and $\Delta t = 0.02\text{s}$ using Forward Time Central Space (FTCS) finite-difference.

(12 marks)

(b) An electromagnetic field is governed the wave equation, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ with the boundary condition $u(0,t) = u(1,t) = 0$ for $0 \leq t \leq 0.2$ and the initial conditions $u(x,0) = \sin(\pi x)$, and $\frac{\partial u}{\partial t}(x,0) = 0$ for $0 \leq x \leq 1$. By taking $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$, find the electromagnetic field using the explicit finite-difference method

(13 marks)

-END OF QUESTIONS –

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FORMULAS

First Order Numerical differentiation:

2-point forward difference

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

2-point backward difference

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

3-point central difference

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

3-point forward difference

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$$

3-point backward difference

$$f'(x) \approx \frac{f(x-2h) - 4f(x-h) + 3f(x)}{2h}$$

5-point central difference

$$f'(x) \approx \frac{f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h))}{12h}$$

Second Order Numerical differentiation:

3-point central difference formula (second derivative)

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

5-point formula for second derivative

$$f'(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h))}{12h^2}$$

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Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Numerical Integration:

Trapezoidal rule:

$$\int_b^b f(x)d(x) \approx \frac{h}{2}\left[(f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right]$$

Simpson's $\frac{1}{3}$ rule:

$$\int_b^b f(x)d(x) \approx \frac{h}{3}\left[(f_0 + f_n + 4\sum_{i=1}^{n/2} f_{2i-1} + 2\sum_{i=1}^{(n/2)-1} f_{2i}\right]$$

Simpson's $\frac{3}{8}$ rule:

$$\int_b^b f(x)d(x) \approx \frac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{(n/3)-1} f_{3i}\right]$$