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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ACTUARIAL MATHEMATICS II
COURSE CODE	:	BWA 31503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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Q1 (a) Cash value is the amount available in cash upon cancellation of an insurance policy.

(i) Identify **THREE (3)** common insurance options that use the policy's net cash values.

(3 marks)

(ii) Discuss the similarities or differences among these three insurance options given in **Q1(a)(i)**.

(7 marks)

(b) For a fully discrete whole life insurance of 1,000 on (x), you are given the following:

- $G = 200$ is the gross premium.
- $e_k = 10,\ k = 1,2,3,...$ is the per policy expense at the start of year k.
- $c_k = 0.04,\ k = 1,2,3,...$ is the fraction of premium at the start of year k.
- $i = 0.05$.
- ${}_{10}CV = 1700$ is the cash value payable upon withdrawal at the end of year 4.
- $q_{x+10}^{(d)} = 0.02$ is the probability of decrement by death.
- $q_{x+10}^{(w)} = 0.18$ is the probability of decrement by withdrawal. Withdrawals occur at the end of the year.
- ${}_{10}AS = 1600$ is the asset share at the end of year 10.

(i) Determine the value of asset share at the end of year 11.

(4 marks)

(ii) If the probability of withdrawal and all expenses for year 11 are each 120% of the values shown above, determine the revised value of the asset share at the end of year 11.

(4 marks)

(iii) Based on **Q1(a)(ii)**, how much does the asset share at the end of the year decreases?

(2 marks)

Q2 (a) Consider an insurance portfolio that will produce zero, one, two, or three claims in a fixed time period with probabilities 0.1, 0.3, 0.4, and 0.2, respectively. An individual claim will be of amount 1, 2, or 3 with probabilities 0.5, 0.4, and 0.1 respectively.

(i) Construct a table and compute $f_s(x) = \Pr(S = s)$ for $x = 0,1,2,3,4$.

(10 marks)

(ii) Calculate the mean $E[N]$ and variance $Var(N)$ of the number of claims.

(4 marks)

(b) Suppose that the claim amount distribution is the same as in **Q2(a).** The distribution of N follows Poisson distribution. The formula for the expectation of S given by

$$E[S] = \lambda E[X] = \lambda p_1,$$

and the variance of S

$$Var(S) = \lambda E\left[X^2\right] = \lambda p_2.$$

Use these formulas and $E[N]$ from **Q2(a)(ii)** to compute $E[S]$ and $Var(S)$.

(6 marks)

Q3 (a) Consider a policy issued at age 35 with an initial gross premium of 1,000 and initial benefit of 120,000. Use the Illustrative Life Table in **Table Q3(a)** with 6% interest to analyze the excess first-year expense allowance and the fifth-year reserve.

Table Q3(a): Illustrative Life Table

Age	l_x	d_x	$1{,}000q_x$
30	95 013.79	145.2682	1.5289
31	94 868.53	152.6317	1.6089
32	94 715.89	160.6896	1.6965
33	94 555.20	169.5052	1.7927
34	94 385.70	179.1475	1.8980
35	94 206.55	189.6914	2.0136
36	94 016.86	201.2179	2.1402
37	93 815.64	213.8149	2.2791
38	93 601.83	227.5775	2.4313
39	93 374.25	242.6085	2.5982
40	93 131.64	259.0186	2.7812
41	92 872.62	276.9271	2.9818
42	92 595.70	296.4623	3.2017
43	92 299.23	317.7619	3.4427
44	91 981.47	340.9730	3.7070
45	91 640.50	366.2529	3.9966

(10 marks)

(b) Five years after issue, the policyholder in **Q3(a)** wishes to change the policy to Term Insurance to age 65 with a coverage of 150,000. Determine the contract premium after the change.

(10 marks)

Q4 (a) Ahmad's age is 50.5 at the valuation date. He receives RM6,000 in salary in the month to the valuation date. Ahmad's salary increases yearly on 1 January and he is planning to retire at age 65. Assume the replacement ratio is 65% and the valuation date of 1 September. Using **Table Q4(a),**

Table Q4(a): Hypothetical Salary Scale

Age	s_x	x	s_x
30	1.00	50	3.41
31	1.06	51	3.63
32	1.13	52	3.86
33	1.20	53	4.10
34	1.28	54	4.35
35	1.36	55	4.62
36	1.44	56	4.91
37	1.54	57	5.21
38	1.63	58	5.53
39	1.74	59	5.86
40	1.85	60	6.21
41	1.96	61	6.56
42	20.9	62	6.93
43	2.22	63	7.31
44	2.36	64	7.70
45	2.51	65	8.08
46	2.67	66	8.48
47	2.84	67	8.91
48	3.02	68	9.35
49	3.21	69	9.82

(i) determine the salary that Ahmad receives over the year of age $\left(49\frac{5}{6}, 50\frac{5}{6}\right)$,

(1 mark)

(ii) calculate the expected salary in Ahmad's final year of work,

(5 marks)

(iii) calculate Ahmad's target pension benefit per year.

(3 marks)

(b) Suppose Fasha, aged 30, is a newly hired employee of DRB Group. She receives RM80,000 in her first year of service at the company. Assuming

- Fasha'sha salary increases 3% per year,
- she receives merit increases of 5% at each of the first three (3) employment anniversaries,
- the pension benefit formula is 1% of the final five (5) year average salary per year of service.

(i) If Fasha retires at age 65, predict the projected final **FIVE (5)** year average salary.

(5 marks)

(ii) Forecast the projected pension benefit Fasha will receive at age 65.

(2 marks)

(iii) Compute the employee's replacement ratio.

(4 marks)

Q5 A Lexis diagram provides a convenient way of showing the relationship between periods and cohorts. Demographic events can be viewed either by calendar time, age or cohort.

(a) Using the Lexis diagram in **Figure Q5(a)**, calculate

(i) the age difference between the oldest and the youngest employees at time -25,

(3 marks)

(ii) the number of employees who have attained age 35 while active in the workforce,

(2 marks)

(iii) the number of employees at time -25, who have attained or will attain age 50 while in the workforce.

(2 marks)

(b) The generation force of mortality at age x for those born at time u is denoted by

$$\mu(x,u) = -\frac{1}{l(x,u)}\frac{\partial}{\partial x}l(x,u).$$

From **Figure Q5(b)**, use the double integral method to show that the number of lives that will attain age x_0 between times t_0 and $t_0 + 1$ and die before time $t_0 + 3$ is given by

$$\int_{t_0}^{t_0+1} l(x_0, y - x_0)\,dy - \int_{x_0+2}^{x_0+3} l(w, t_0 + 3 - w)\,dw.$$

(8 marks)

(c) A population density function is defined by

$$l(x,u) = b(u)\,s(x,u).$$

Let

$$b(u) = 100\left[1 - e^{-u/100}\right] \quad u > 0$$
$$s(x) = e^{-x/100} \quad x > 0.$$

Calculate the number of individuals between ages 25 and 50 at time 100.

(5 marks)

-END OF QUESTIONS-

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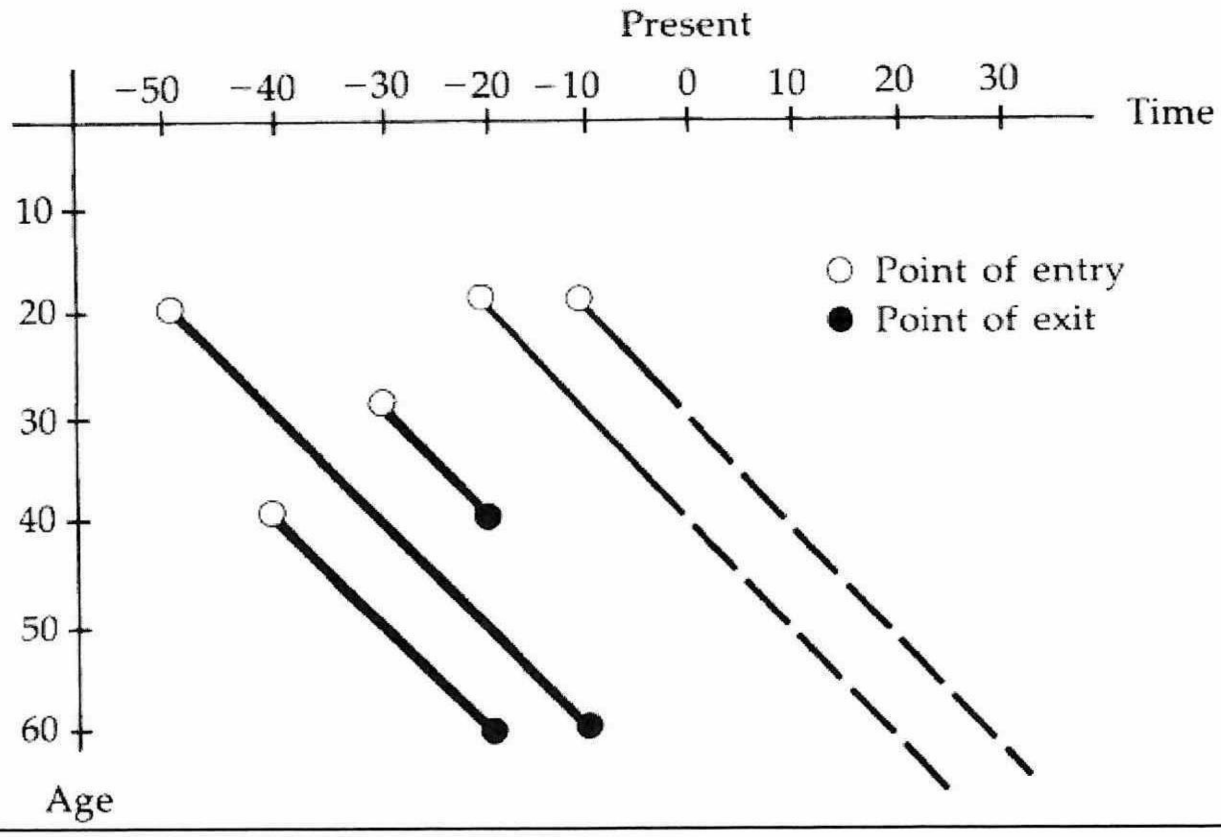

Figure Q5(a)

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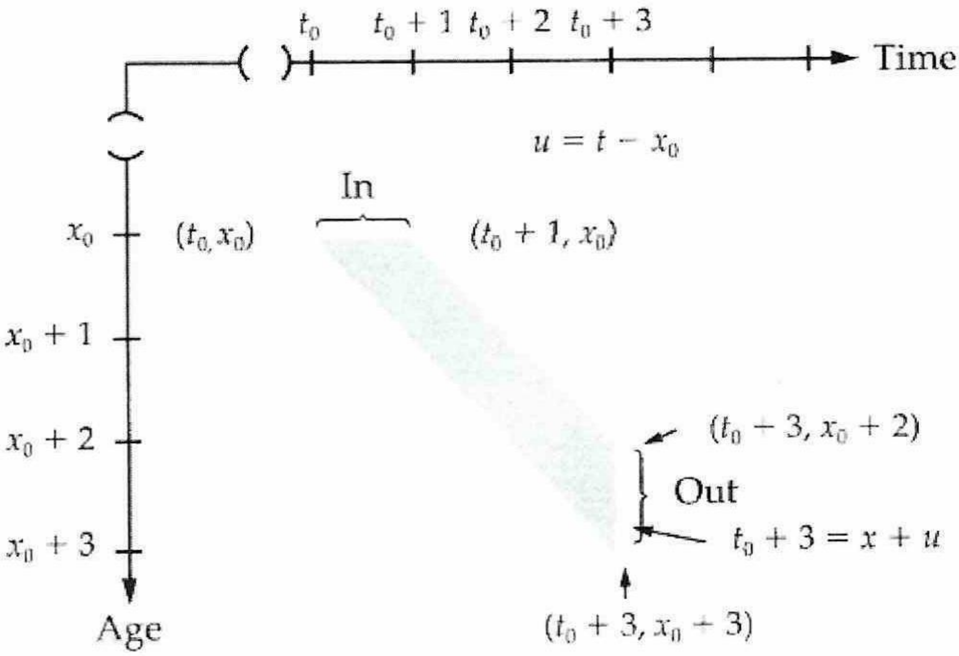

Figure Q5(b)

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FORMULAE

$$\Pr(N=n)=\frac{\lambda^n e^{-\lambda}}{n!}$$

$$\ddot{a}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^k\cdot {}_kp_x$$

$$A^{1}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^{k+1}\cdot {}_kp_x\cdot q_{x+k}$$

$${}_nE_x=v^n\,{}_nP_x$$

$$v^n=\frac{1}{(1+i)^n}$$

$${}_kP_x=\frac{l_{40+k}}{l_{40}}$$

$$-{}_0V=P-\upsilon q_x b$$

$${}_kV=\frac{{}_0V+P\ddot{a}_{x:\overline{k}|}-bA^{1}_{x:\overline{k}|}}{{}_kE_x}$$

$${}_{k+g}V'=\frac{{}_kV'+P'\ddot{a}_{x+k:\overline{g}|}-b'A^{1}_{x+k:\overline{g}|}}{{}_gE_{x+k}}$$

$$\int_{t_0}^{t_1} l(x,t-x)\,dt$$

$$\int_{x_0}^{x_1} l(x,t_0-x)\,dx$$