UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHODS FOR FLUID DYNAMICS
COURSE CODE	:	BWA 33203
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS

1. ANSWER ALL QUESTIONS
2. THIS FINAL EXAMINATION IS CONDUCTED VIA
   ☐ Open book
   ☒ Closed book
3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 4^{th} order Runge-Kutta method for $\frac{dy}{dx} = f(x, y)$ is given as follows:

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4),$$

where

$$k_1 = f(x_i, y_i),$$

$$k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h),$$

$$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h),$$

$$k_4 = f(x_i + h, y_i + k_3 h).$$

Hence, write the formula of 4^{th} order Runge-Kutta for:

(a) $\frac{dy_1}{dx} = f_1(x, y_1, y_2)$, $\frac{dy_2}{dx} = f_2(x, y_1, y_2)$.

(6 marks)

(b) $\frac{dy_1}{dx} = f_1(x, y_1, y_2, y_3)$, $\frac{dy_2}{dx} = f_2(x, y_1, y_2, y_3)$, $\frac{dy_3}{dx} = f_3(x, y_1, y_2, y_3)$.

(7 marks)

(c) Hence, solve the following set of differential equations using 4^{th} order Runge-Kutta by assuming that $y_1(1) = 4$ and $y_2(1) = 6$. Integrate to $x = 1.5$ with $\Delta x = 0.5$ (use 4 decimal places).

$$\frac{dy_1}{dx} = y_1,$$

$$\frac{dy_2}{dx} = 4 - y_2 + y_1.$$

(11 marks)

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Q2 Consider

$$I = \int_a^b g(x)dx \, ,$$

where

$$g(x) = \int_c^d (x^2 + y)dy \, .$$

(a) By using trapezoidal rule to calculate I, the following table is obtained.

Table Q2.1

i	x_i	y_i	$g(x_i)$
0	1.0		3.50
1	1.5		4.75
2	2.0	2.0	6.50
3	2.5	2.5	8.75
4	3.0	3.0	11.50
5	3.5		14.75
6	4.0		18.50

From **Table Q2.1**:

(i) Determine the value of a, b, c, d, Δx and Δy.

(6 marks)

(ii) Show the calculation of any value of $g(x)$. (Choose only one value).

(6 marks)

(iii) Calculate I using trapezoidal rule.

(3 marks)

(b) From **Q2(a)(i)** calculate I using 2-point Gauss quadrature.

(10 marks)

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Q3 Given two dimensional second-order partial differential equation (PDE) with dependent variable ϕ and independent variables x and y. Consider a, b, c, d, e, f and g are constants.

(a) Hence, write the general form of PDE.

(2 marks)

(b) Classify the three types of the PDE with one example(s) for each type. Write the example(s) from the following options:

(i) Heat equation.

(ii) Wave equation.

(iii) Laplacian equation.

(iv) Poisson's equation.

(10 marks)

Q4 Assume $T(x, y)$ is a temperature of a heated plate in the form of Laplacian equation with Dirichlet boundary conditions as follows:

$$T(x,0) = 0,\ T(x,1) = 100 \text{ at } 0 \le x \le 1,$$

$$T(0, y) = 80,\ T(1, y) = 60 \text{ at } 0 \le y \le 1.$$

(a) Write the related equation.

(1 marks)

(b) Sketch the geometrical configuration of the problem.

(2 marks)

(c) Apply the central differences to discretise **Q4(a)** and **Q4(b)**. Use four uniform subintervals in the x and y directions to form a matrix $A_{9\times 9}T_{9\times 1} = B_{9\times 1}$. (Without a solution).

(16 marks)

- END OF QUESTIONS -

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APPENDIX A

1. Trapezoidal rule for $\int_a^b f(x)dx \approx \frac{\Delta x}{2}\left\{ f_0 + \sum_{i=1}^{n-1} f_i + f_n \right\}$

2. 2-point Gauss quadrature for $\int_{-1}^{1} f(x)dx \approx f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

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