Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2022/2023

| | | |
|---|---|---|
| COURSE NAME | : | DISCRETE MATHEMATICS |
| COURSE CODE | : | BWA 10603 |
| PROGRAMME CODE | : | BWA |
| EXAMINATION DATE | : | JULY/ AUGUST 2023 |
| DURATION | : | 2 HOURS 30 MINUTES |
| INSTRUCTIONS | : | 1. ANSWER ALL QUESTIONS |
| | | 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** |
| | | 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

**Q1** Given a proposition: $P(n): 1+4+7+\ldots+3n-2=\frac{n(3n-1)}{2}$ for all $n \in \boldsymbol{N}$. Prove the proposition above by using mathematical induction.

(10 marks)

**Q2** Test the validity of the following argument:

If I study, then I will not fail mathematics.
If I do not play basketball, then I will study.
But I failed mathematics.

---

Therefore I must have played basketball.

(10 marks)

**Q3** (a) Consider the relation $R = \{(a,\ a), (a,\ b), (b,\ c), (c,\ c)\}$, on the set $A = \{a,\ b,\ c\}$. Find:

(i) reflexive $(R)$,
(ii) symmetric $(R)$,
(iii) transitive $(R)$.

(6 marks)

(b) Let $\ell$ be any collection of sets. Determine whether the relation of set inclusion $\subseteq$ a partial order on $\ell$.

(4 marks)

**Q4** (a) Let $V = \{1,\ 2.\ 3.\ 4\}$. For the following functions $f: V \to V$ and $g: V \to V$, $f = \{(1,\ 3), (2,\ 1), (3,\ 4), (4,\ 3)\}$, and $g = \{(1,\ 2), (2,\ 3), (3,\ 1), (4,\ 1)\}$.

(i) Determine whether $f$ is invertible. Give the reason for your answer.
(ii) Find $f \circ g$.

(4 marks)

(b) Find

(i) 29 (mod 6),
(ii) $-555$ (mod 11).

(4 marks)

(c) Let $f: \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 5x - 4$. Now $f$ is one-to-one and onto; hence $f$ has an inverse function $f^{-1}$. Find a formula for $f^{-1}$.

(2 marks)

**Q5** Given polynomial $f(x) = 7x^5 - 4x^4 + 2x^3 - 7x^2 + 3x - 11$.

(a) Determine the total number of operation required if using direct method.

(3 marks)

(b) Determine the total number of operation required if using synthetic division.

(2 marks)

(c) Calculate $f(-2)$ by using synthetic division.

(5 marks)

**Q6** Given two polynomials over $\mathbf{Z}_7$

$$f(x) = 6x^3 - 5x^2 + 2x - 4,$$
$$g(x) = 5x^3 + 2x^2 + 6x - 1,$$

find

(a) $f(x) + g(x)$.

(4 marks)

(b) $f(x)g(x)$.

(6 marks)

**Q7** (a) Find the minimum number of students needed to guarantee that 4 of them were born:

(i) on the same day of the week;

(ii) in the same month.

(4 marks)

(b) In a class of 30 students, 10 got $A$ on the first test, 9 got $A$ on a second test, and 15 did not get an $A$ on either test. Find the number of students who got an $A$ on both tests.

(3 marks)

(c) Find the number of ways a coin can be tossed 6 times so that there is exactly 3 heads and no two heads occur in a row.

(3 marks)

**Q8** Consider the third-order homogeneous recurrence relation,

$$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}.$$

(a) Find the general solution.

(3 marks)

(b) Find the solution with initial conditions $a_0 = 3,\ a_1 = 4,\ a_2 = 12$.

(7 marks)

**– END OF QUESTIONS –**