UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	VECTOR CALCULUS
COURSE CODE	:	BWA 20803
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) **Figure Q1(a)** illustrates a curve in two-space. Evaluate $\int_C 2xy\,dx+\left(x^2+y^2\right)dy$ along the curve C shown in the **Figure Q1(a)**.

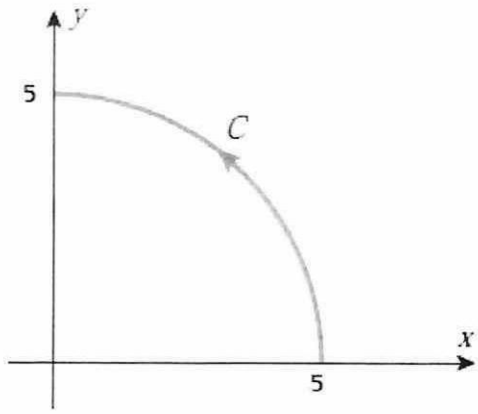

Figure Q1(a)

(7 marks)

(b) Sketch the surface area of solid region bounded by the closed surface of the finite cylinder $x^2+y^2=2$ and the planes $x+z=2$ and $z=0$.

(3 marks)

(c) Use the Divergence theorem to evaluate $\iint_S \hat{F}\cdot\hat{n}\,dS$, where $\hat{F}=\left\langle x^3, y^3, 6z\right\rangle$ and S is the closed surface of the finite cylinder $x^2+y^2=2$ and the planes $x+z=2$ and $z=0$. Take $\hat{n}$ to be the outward unit normal.

(10 marks)

Q2 (a) The equation of a curve is given by $x=t-\frac{1}{2}t^2$, $y=t^2$, $z=t+\frac{1}{2}t^2$. Find

(i) Unit tangent vector **T**.

(3 marks)

(ii) Unit binormal vector **B**.

(3 marks)

(iii) The curvature K.

(3 marks)

(b) Determine the component functions of $\hat{r}(t)=\left\langle \sin^2(t)+1, t+e^t, 6t^4\right\rangle$.

(2 marks)

(c) If $\hat{r}(t)=\left\langle \sin^2(2t), \ln\left(t^2\right)+e^{2t}, 6t^4 \right\rangle$, calculate the vector $\hat{r}'(t_0)$ when $t_0=\dfrac{\pi}{2}$.

(3 marks)

(d) Verify the identity $\dfrac{d}{dt}\left(\hat{F}\times\hat{G}\right)=\hat{F}\times\dfrac{d\hat{G}}{dt}+\dfrac{d\hat{F}}{dt}\times\hat{G}$, if $\hat{F}=\left\langle 0, -2e^{2t}, 6t^2 \right\rangle$ and $\hat{G}=\left\langle \sin(2t), 0, t+1 \right\rangle$.

(6 marks)

Q3 (a) Verify Stokes' theorem for the vector field $\hat{F}=\left\langle 4y, -2x, z^3 \right\rangle$ and S is the portion of the cone $z=\sqrt{x^2+y^2}$ bounded by the plane $z=2$. Take $\hat{n}$ to be the outward unit normal.

(12 marks)

(b) Evaluate $\int_C \left[y-\sin(x)\right]dx+\cos(x)\,dy$, where c is the perimeter of the triangle formed by the lines $y=0$, $x=\dfrac{\pi}{2}$, and $y=\dfrac{2x}{\pi}$ using Green's theorem.

(8 marks)

Q4 (a) Given an equation $\phi(x,y,z)=3x^3y^2\ln z$. Calculate $\nabla\cdot\nabla\phi$.

(4 marks)

(b) Show that $\nabla\cdot\nabla\phi=\nabla^2\phi$ where ϕ is differentiable vector function of x, y and z.

(3 marks)

(c) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2+3=z$ at the point $(2,-1,2)$.

(5 marks)

(d) If $\hat{B}=\left\langle z\ln(x), -2x^2\cos(2y), 2zx \right\rangle$, calculate curl curl $\hat{B}$.

(3 marks)

(e) Suppose that a particle travels along a circular helix in 3-space so that its position vector $\hat{r}(t)=\left\langle 4\cos(\pi t), 4\sin(\pi t), t \right\rangle$. Compute the displacement and distance travelled by the particle during the time interval $1\le t\le 3$.

(5 marks)

- END OF QUESTIONS -

APPENDIX A

Formula:

$$\mathbf{T}(t)=\frac{d\mathbf{r}/dt}{|d\mathbf{r}/dt|} \qquad \mathbf{N}=\frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}$$

$$\kappa=\frac{|d\mathbf{T}/dt|}{|d\mathbf{r}/dt|} \quad or \quad \kappa=\frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\int_C \xi(x,y,z)\,ds=\int_a^b \xi(x,y,z)\sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$

s denotes the arc length,

$$s=\int_C ds=\int_a^b \sqrt{[x'(t)]^2+[y'(t)]^2+[z'(t)]^2}\,dt$$

$$\int_C P(x,y,z)\,dx=\int_a^b P(x(t),y(t),z(t))\,x'(t)\,dt$$

$$\int_C Q(x,y,z)\,dx=\int_a^b Q(x(t),y(t),z(t))\,y'(t)\,dt$$

$$\int_C R(x,y,z)\,dx=\int_a^b R(x(t),y(t),z(t))\,z'(t)\,dt$$

$$\int_C \hat{F}\cdot d\hat{r}=\iint_s \left(\nabla\times\hat{F}\right)\cdot\hat{n}\,dS$$

where $d\hat{r}=\langle dx,dy,dz\rangle$ and $\hat{F}(x,y,z)=\langle P(x,y,z),P(x,y,z),P(x,y,z)\rangle$

$$\int_C P(x,y)dx+Q(x,y)dy=\iint_R\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$$

$\nabla\phi_1\cdot\nabla\phi_2=\|\nabla\phi_1\|\|\nabla\phi_2\|\cos\theta$ where ϕ_1, ϕ_2 is differentiable vector functions of x, y and z.