CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHODS
COURSE CODE	:	BEE 32402
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

CONFIDENTIAL

Q1 In pharmaceutical manufacturing, Near-Infrared Spectroscopy (NIR) serves as a process analysis technology, allowing for the real-time, nondestructive, and noncontact measurement of Critical Material Attributes (CMAs) during manufacturing processes. This method utilizes the electromagnetic spectrum ranging from 780 nm to 2500 nm in multiple non-destructive measurement applications. **Table Q1** presents the tabulated data of the measured signal at five (5) specific wavelengths. To enhance the quality of NIRS signals, the computation of the rate of change can effectively eliminate undesired signals.

Table Q1

Wavelength, x (nm)	750	900	950	1000	1150
Measured Signal, E(x)	17.41	95.22	332.12	455.74	429.41

By applying all appropriate numerical first-order derivative methods to the following questions:

(a) Estimate the rate of change of the signal at a wavelength of 950 nm using a 50 nm interval.

(9.5 marks)

(b) Estimate the rate of change of the signal at a wavelength of 950 nm by utilizing a 200 nm interval.

(9.5 marks)

(c) Identify the best method in estimating the rate of change of the signals with a concise justification if the exact solution is 2.4566.

(6 marks)

Q2 (a) The output power of a system can be represented by a continuous function $f(t)$ over the interval $a \leq t \leq b$ that is given by,

$$f(t) = \int_{1}^{3} 2t^3 + \ln(t^2 + 2)\, dx$$

(i) Approximate the continuous function $f(t)$, by using trapezoidal rule with subintervals, n=8 and n=15.

(9 marks)

(ii) Find the absolute error for each subinterval from **Q2(a)(i)**.

(3 marks)

(iii) Determine which method approximates better.

(1 mark)

(b) A 50 Ω load resistor is observed to have a continuous current flow from 0 to 1.5 seconds. The current flow can be denoted by function,

$$i(t) = \int_0^{1.5} \frac{e^{2t} - e^{-2t}}{3}\, dt$$

Approximate the absolute error of $i(t)$ when the Simpson's Rule with 15 subintervals is used.

(12 marks)

Q3 (a) An armadillo was found dead in the woods by a ranger, which he assumed was shot by a poacher. The recorded body temperature of the carcass was 29 °C (degree Celcius) while the temperature of the woods was assumed to be uniform at 25 °C. The rate of cooling of the body can be expressed as:

$$\frac{dT}{dt} = -k(T - T_a),$$

where T is the temperature of the body in °C, T_a is temperature of the surrounding forest (in °C) and k is proportionally constant. Let initial temperature of the cougar be 36 °C while $k = 0.174$.

i Estimate the temperature of the carcass at time, $t = 7$ hours by using Euler's method with $\Delta t = 1$ hour.

(9 marks)

ii. Approximate how long the cougar had been killed at $T = 28.5$ °C by using any appropriate interpolation techniques.

(4 marks)

(b) Solve $y'' + y = 0, y(0) = 3, y(1) = -3$ by using finite-difference method with $h = 0.2$.

(12 marks)

Q4 (a) Given the heat equation $\frac{\partial u}{\partial t} = 3^2 \frac{\partial u^2}{\partial x^2}$, $0 \le x \le 6, t > 0$ with the boundary conditions distribution $u(0,t) = 0$ and $u(6,t) = 1.5$ for $t > 0$ and the initial condition, $u(x,0) = x(1 - 0.125x)$. Find $u(x, 0.002), u(x, 0.004)$ and $u(x, 0.006)$ using explicit finite-difference method with $\Delta x = 2$.

(12 marks)

(b) An elastic string which is fixed at both ends is governed by the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = 2\frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, \quad t \ge 0,$$

where $u(x,t)$ is the displacement of the string. The initial conditions are given by

$$u(x,0) = sin(3\pi x),$$

$$\frac{\partial u}{\partial t}(x,0) = 0.25, \quad 0 \le x \le 1$$

with the boundary condition

$$u(0,t) = u(1,t) = 2t \quad 0 \le t \le 0.4$$

Determine the variation of the displacement of the string by using the explicit difference method for using $h = \Delta x = 0.25$ and $k = \Delta t = 0.2$.

(13 marks)

-END OF QUESTIONS –

FINAL EXAMINATION

SEMESTER / SESSION : SEM II 2022/2023
COURSE NAME : NUMERICAL METHODS
PROGRAMME CODE : BEJ/BEV
COURSE CODE : BEE32402

FORMULAS

First Order Numerical differentiation:

2-point forward difference

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

2-point backward difference

$$f'(x) \approx \frac{f(x)-f(x-h)}{h}$$

3-point central difference

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

3-point forward difference

$$f'(x) \approx \frac{-3f(x)+4f(x+h)-f(x+2h)}{2h}$$

3-point backward difference

$$f'(x) \approx \frac{f(x-2h)-4f(x-h)+3f(x)}{2h}$$

5-point central difference

$$f'(x) \approx \frac{f(x-2h)-8f(x-h)+8f(x+h)-f(x+2h))}{12h}$$

Second Order Numerical differentiation:

3-point central difference formula (second derivative)

$$f''(x) \approx \frac{f(x-h)-2f(x)+f(x+h)}{h^2}$$

5-point formula for second derivative

$$f'(x) \approx \frac{-f(x-2h)+16f(x-h)-30f(x)+16f(x+h)-f(x+2h))}{12h^2}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM II 2022/2023 PROGRAMME CODE : BEJ/BEV
COURSE NAME : NUMERICAL METHODS COURSE CODE : BEE32402

Boundary value problems:

Finite difference method:

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h} \qquad y_i'' \approx \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$$

Numerical Integration:

Trapezoidal rule:

$$\int_b^b f(x)d(x) \approx \frac{h}{2}\left[(f_0 + f_n + 2\sum_{i=1}^{n-1} f_i\right]$$

Simpson's $\frac{1}{3}$ rule:

$$\int_b^b f(x)d(x) \approx \frac{h}{3}\left[(f_0 + f_n + 4\sum_{i=1}^{n/2} f_{2i-1} + 2\sum_{i=1}^{(n/2)-1} f_{2i}\right]$$

Simpson's $\frac{3}{8}$ rule:

$$\int_b^b f(x)d(x) \approx \frac{3h}{8}\left[f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{(n/3)-1} f_{3i}\right]$$