UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHOD
COURSE CODE	:	BDA 34103
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A**. 2. ANSWER **TWO (2)** QUESTIONS ONLY IN **PART B**. 3. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 4. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 Given matrix $A = \begin{pmatrix} 2 & 2 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 5 \end{pmatrix}$.

(a) Find the eigenvalues using characteristic equation. Perform your calculation in 4 decimal points.

(7 marks)

(b) Find the largest eigenvalue and the corresponding eigenvector using Power Method Method. Given the initial eigenvector $v^{(0)} = (1 \quad 0 \quad 1)^T$. Iterate from $k = 0$ to $k = 6$. Perform your calculation in 4 decimal points.

(10 marks)

(c) Compare the obtained largest eigenvalue to the answer from the characteristic equation in terms of absolute error.

(3 marks)

Q2 The rate of heat flow (conduction) between two points on a cylinder heated at one end is given by

$$\frac{dQ}{dt} = \lambda A \frac{dT}{dx}$$

where λ is a constant, A is the cylinder's cross-sectional area, Q is heat flow, T is temperature, t is time and r is the distance from the heated end.

Given that

$$\frac{dT}{dx} = \frac{100(L-x)(20-t)}{100-xt}$$

where L is the length of the rod.

If the initial condition is $Q(0)=0$ and the parameters A is 10 where L= 2A, x= 2.5 cm and λ = 0.4 cal cm/s. Calculate the heat flow for t = 0 s to 35 s using Heun's method. Use h= 5.

(20 marks)

Q3 The heat transfer performance of a new conductor bar of length 20 cm is under inspection. The material is fully insulated such that the heat transfer is only one dimensional in axial direction (x-axis). The left end is maintained at temperature of 100°C, while the right end is maintained at temperature of 20°C, for $t > 0$. The distribution of the initial temperatures is shown in **Figure Q3.** The unsteady state heat conduction equation is given by;

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial x^2} = 0$$

Where κ is a thermal diffusivity of material and x is the longitudinal coordinate of the bar. The thermal diffusivity of the material is given as $\kappa = 10$ cm²/s, and $\Delta t = 2$ second.

(a) By considering numerical differentiation $\frac{\partial^2 T}{\partial x^2} = \frac{T(x+1,t) - 2T(x,t) + T(x-1,t)}{\Delta x^2}$ and $\frac{\partial T}{\partial t} = \frac{T(x,t+1) - T(x,t)}{\Delta t}$, deduce that the temperature distribution along the bar at point $(x, t+1)$ in explicit finite-difference form is given by

$$T(x,t+1) = 0.8T(x-1,t) - 0.6T(x,t) + 0.8T(x+1,t)$$

(6 marks)

(b) Draw the finite difference grid to predict the temperature of all points up to 4 seconds. Label all unknown temperatures on the grid.

(6 marks)

(c) Determine all the unknown temperatures.

(4 marks)

(d) Analyze why the heat conduction in this question is more suitable to be solved using the implicit finite-difference approach.

(4 marks)

PART B: ANSWER TWO (2) QUESTIONS ONLY

Q4 Mechanical engineers sometimes compute the trajectories of projectiles like rockets. A related problem deals with the trajectory of a thrown ball. The trajectory of a ball is defined by the (x, y) coordinates, as shown in **Figure Q4**. The trajectory can be modeled as:

$$y = \tan(\theta_0)x - \frac{g}{2v_0^2 \cos^2 \theta_0}x^2 + y_0$$

(a) Find the appropriate initial angle θ_0 using Secant method, if the initial velocity v_o = 20 *m/s* and the catcher x = 40 m. Note that the ball leaves the thrower's hand at an elevation of $y_0 = 1.8$ m and the catcher receives it at 1 m. Use a value of 9.81 m/s^2 for g. Perform your calculation until 6-th iteration with starting interval at 0 to $\frac{\pi}{4}$. All data /values shall be in 4-decimal place.

(14 marks)

(b) Analyze why Secant method is more suited than Newton Raphson method for this problem.

(3 marks)

(c) By using the same condition as in part (a), analyze whether is Bisection method is more suited than Secant method for this problem.

(3 mark)

Q5 **Figure Q5** shows a spring-mass system composed of three masses suspended vertically by a series of springs. To develop a mathematical model of the spring-mass system, Newton's second law can be employed in conjunction with force balances. For each mass, the Newton's second law can be expressed as:

$$m\frac{d^2x}{dt^2} = F_D - F_U$$

where m is the mass of an object, $\frac{d^2x}{dt^2}$ is the acceleration of an object, F_D is downward force and F_U is upward force. When the system eventually comes to rest (steady state), the displacements of the masses are expressed as

$$\begin{aligned} 3kx_1 - 2kx_2 &= m_1 g \\ -2kx_1 + 3kx_2 - kx_3 &= m_2 g \\ - kx_2 + kx_3 &= m_3 g \end{aligned}$$

(a) Write the system of linear equations in complete matrix form.

(2 marks)

(b) Solve the system of linear equations in **Q5(a)** using Doolittle method. Given $k = 10\text{kg/s}^2$, $g = 9.81\text{m/s}^2$, $m_1 = 20\,kg$, $m_2 = 250\,kg$ and $m_3 = 50\,kg$.

(8 marks)

(c) Compare the answer in **Q5(b)** with Crout method. Give conclusion to your finding.

(10 marks)

Q6 (a) Given the following data which is generated by the function $y = \dfrac{1}{3x^2}$:

Table Q6(b)

x	1	2	3	4	5
$y = f(x)$	0.33333	0.08333	0.03704	0.02083	0.01333

Comparing quadratic (x within [2,4]) and cubic (x within [2,5]) Lagrange interpolations method, investigate which method gives the highest accuracy (in terms of absolute error) in predicting the value of $f(3.5)$.

(10 marks)

(b) The movement of an object is represented by

$$s(t) = 8e^{-0.5t}$$

Utilize numerical differentiation using 3 point central formula with $h = 1$ and $h = 0.5$ to approximate the object's velocity and acceleration at 15 second. State which h gives better approximation. Correct to 4 decimal places.

(10 marks)

-END OF QUESTION-

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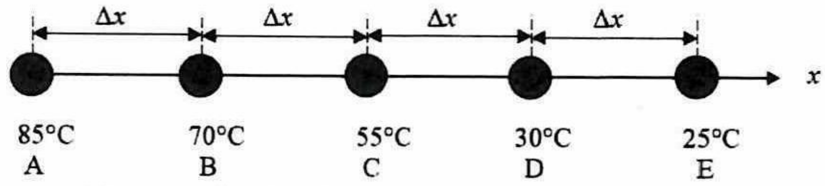


Figure Q3: Distribution of the initial temperatures

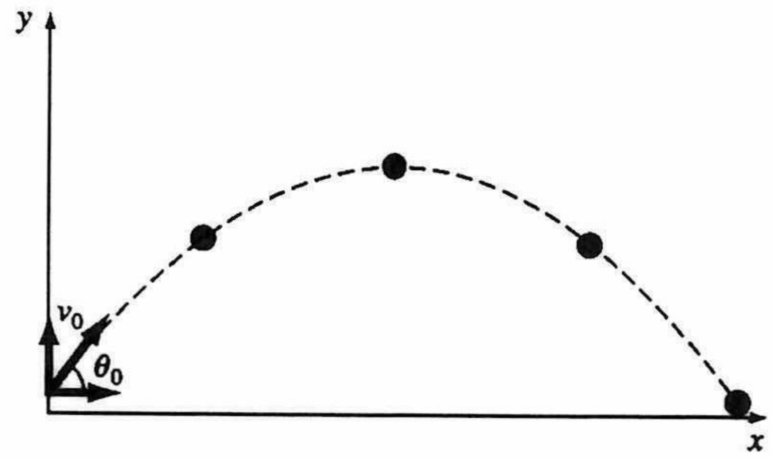
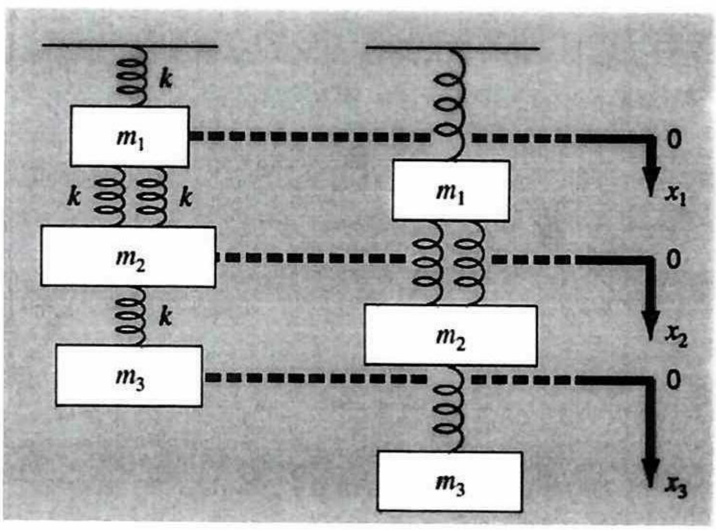


Figure Q4



The system before release The system after release

Figure Q5

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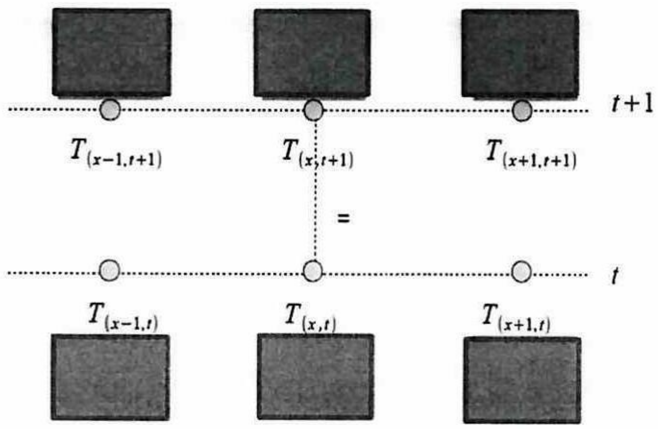
FORMULA

Implicit Crank Nicolson Method:



Euler 's Method:

$$y(x_{i+1}) = y(x_i) + y'(x_i)\, h$$

Power Method:

$$\{V\}^{k+1} = \frac{[A]\{V\}^k}{\lambda_{k+1}}$$

Inverse Power Method:

$$\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$$

Simpson 3/8:

$$\int_{x_1}^{x_n} y(x)dx = \frac{3h}{8}\left(y_1 + 3\,(y_2 + y_3 + y_5 + y_6 + \cdots) + 2(y_4 + y_7 + \cdots) + y_n\right)$$

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FORMULA

Gauss Quadrature:

$$x_\xi = \frac{1}{2}[(1-\xi)x_0 + (1+\xi)x_n]$$
$$I = \left(\frac{x_n - x_0}{2}\right) I_\xi$$
$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + \cdots + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692	0.555555556
	0.0	0.888888889

3 Point Central Difference:

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h}$$
$$y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2}$$

Bisection Method:

$$c = \frac{a+b}{2}$$

Secant Method:

$$x_{i+1} = \frac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$$

Newton Divided Difference:

$$y(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$