Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY I
COURSE CODE	:	BDJ12203
PROGRAMME CODE	:	BDJ
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 (a) Evaluate the following limits.

(i) $\lim_{x \to h} \dfrac{x^2 - 2hx + h^2}{x - h}$.

(3 marks)

(ii) $\lim_{x \to 0} \dfrac{6}{x}\left(\dfrac{1}{4+x} - \dfrac{1}{4-x}\right)$.

(3 marks)

(iii) $\lim_{x \to \infty} \sqrt{x^8 + 6x^4} - x^4$.

(4 marks)

(iv) $\lim_{\theta \to 0} \dfrac{\tan\theta - \theta}{\theta - \sin\theta}$.

(5 marks)

(b) Compute the values of A and B so that $g(x)$ is continuous everywhere.

$$g(x) = \begin{cases} \sin\left(\dfrac{\pi}{2x}\right) + Ax & , \quad x < 1, \\ x^2 - Bx + 2 & , \quad x = 1, \\ \dfrac{1}{2x-3} & , \quad x > 1. \end{cases}$$

(5 marks)

Q2 (a) Differentiate $f(x) = x^{-2}\sin^2(x^3)$.

(4 marks)

(b) If $y\sin\left(\dfrac{1}{y}\right) = 1 - xy$, calculate $\dfrac{dy}{dx}$ by using implicit differentiation.

(7 marks)

(c) Using logarithmic differentiation, evaluate $\dfrac{dy}{dx}$ for

$$y = \frac{3\sqrt{1-x}}{x^2\left[5x^2 + \cos(2x)\right]^{\frac{3}{2}}}.$$

(9 marks)

Q3 (a) Evaluate $\int \cos^3 x \, dx$.

(7 marks)

(b) By using the substitution $t = \tan\theta$, find the value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2 \, d\theta}{\sin(2\theta)}.$$

(8 marks)

(c) Evaluate the integral of improper fractions below,

$$\int \frac{x^2}{x^2+3x+2} \, dx.$$

(10 marks)

Q4 (a) Determine whether the series converges for $\sum_{n=1}^{\infty} \frac{n}{e^n}$.

(8 marks)

(b) Given $f(x) = e^x$.

(i) Find the Taylor series generated by $f(x) = e^x$ at $x = 0$.

(4 marks)

(ii) What is the special name for the Taylor series when $x = 0$?

(2 marks)

(iii) Use the series for e^x to find the n^{th} series of e^{x^3}.

(5 marks)

(iv) Estimate $\int_0^{0.1} e^{x^3} \, dx$.

(6 marks)

- END OF QUESTIONS -

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Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \csc x \cot x \, dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x \, dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C, \ |x|<1$$

$$\int \frac{-1}{\sqrt{1-x^2}} dx = \cos^{-1} x + C, \ |x|<1$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C$$

$$\int \frac{-1}{1+x^2} dx = \cot^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{x^2-1}} dx = \csc^{-1} x + C, \ |x|>1$$

$$\int \frac{1}{\sqrt{x^2+1}} dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1} x + C, \ |x|>1$$

$$\int \frac{-1}{|x|\sqrt{1-x^2}} dx = \operatorname{sech}^{-1} |x| + C, \ 0<x<1$$

$$\int \frac{-1}{|x|\sqrt{1+x^2}} dx = \operatorname{csch}^{-1} |x| + C, \ x \neq 0$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1} x + C, & |x|<1 \\ \coth^{-1} x + C, & |x|>1 \end{cases}$$

TAYLOR AND MACLAURIN SERIES

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

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TRIGONOMETRIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

TRIGONOMETRIC SUBSTITUTION

$t = \tan\frac{1}{2}x$		$t = \tan x$	
$\sin x = \frac{2t}{1+t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$
$\tan x = \frac{2t}{1-t^2}$	$dx = \frac{2dt}{1+t^2}$	$\tan 2x = \frac{2t}{1-t^2}$	$dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x\cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x\cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$
$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$
$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$	