Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ENGINEERING MATHEMATICS IV
COURSE CODE	:	BDA 34003
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A**. 2. ANSWER **TWO (2)** QUESTION ONLY IN **PART B**. 3. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 4. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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PART A: ANSWER ALL QUESTIONS

Q1 The characteristic equation of a 3 degree of freedom spring mass system as shown in **Figure Q1** can be further developed as a set of simultaneous equations:

$$8V_1 - 6V_2 = 0$$
$$-4V_1 + 2V_2 - V_3 = 0$$
$$-4V_2 + 4V_3 = 0$$

(a) Write the simultaneous equation above in a complete matrix form of $[A][V] = 0$.

(2 marks)

(b) Determine the largest eigenvalue and its corresponding eigenvector using Power Method. Use the initial eigenvector $[V] = (1 \quad 0 \quad 1)^T$ and stop the iteration when $(|\lambda_{i+1} - \lambda_i| < 0.04)$. Perform your calculation in 4 decimal points.

(8 marks)

(c) Determine the smallest eigenvalue and its corresponding eigenvector using Inverse Power Method. Use the initial eigenvector $[V] = (1 \quad 0 \quad 1)^T$ and stop the iteration when $(|\lambda_{i+1} - \lambda_i| < 0.006)$. Perform your calculation in 4 decimal points.

(10 marks)

Q2 The equation of performance for a machine system under an inspection is expressed as:

$$y'' + 3y' = y + x^2$$

Given the boundary condition for the system as $y(0) = 2$ and $y(2) = 5$. As an engineer, you are responsible to check the system's performance at the following point: $x = 0.5, 1$ and 1.5.

(a) By using central finite difference approximation, prove that the differential equation can be written as $y_{i-1} - 9y_i + 7y_{i+1} = x_i^2$.

(7 marks)

(b) Considering the boundary condition, deconstruct the differential equation into a matrix form.

(9 marks)

(c) Solve for the unknown at $x = 0.5, 1$ and 1.5.

(4 marks)

Q3 An insulated composite rod is formed of two parts arranged end to end, and both halves are of equal length. Part A has thermal conductivity, k_A for $0 \le x \le 1/2$, and part B has thermal conductivity k_B for $1/2 \le x \le 1$. The transient heat conduction equations that describe the temperature, T over the length, x of the composite rod are:

$$\frac{\partial T}{\partial t} - \frac{\partial^2 T}{\partial x^2} = 0, \qquad 0 \le x \le \frac{1}{2}$$
$$\frac{\partial T}{\partial t} - 2\frac{\partial^2 T}{\partial x^2} = 0, \qquad \frac{1}{2} < x \le 1$$

where T = temperature, x = axial coordinate and t = time. The boundary condition and initial conditions are given as:

Boundary conditions:	$T(0,t) = 1,$	$T(1,t) = 5$
Initial condition:	$T(x,0) = 0,$	$0 < x < 1$

(a) Draw the finite difference grid to predict the temperature of all points up to 0.02 seconds, if given $\Delta x = 0.25$ and $\Delta t = 0.01$. Label all unknown temperatures on the grid.

(5 marks)

(b) Transform the unsteady state heat conduction equation into a system of linear equations using the explicit finite difference approach.

(12 marks)

(c) Determine the unknown temperatures at $t = 0.02$ second.

(3 marks)

PART B: ANSWER TWO (2) QUESTIONS

Q4 The stress concentration factor, K for a flat bar with a centric hole under axial loading is:

$$K = 3.00 + 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 + 1.53\left(\frac{2r}{D}\right)^3$$

where:

r = radius of the hole, and
D = the width of the bar,

If D = 75 mm, approximate the value of r to obtain K = 3.5.

(a) Determine your result using Secant method. All data /values shall be in 4-decimal place. Do iteration up to five (5) iteration or stop if two consecutive r values are less than 0.001. (Hint: r is somewhere between 4 and 6 mm. Use this value as initial guess).

(9 marks)

(b) Compare your result from **Q4(a)** with Bisection method by using the same condition with **Q4(a).**

(8 marks)

(c) Discuss the differences of the answers from **Q4(a) and Q4(b).**

(3 marks)

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Q5 (a) Solve the initial value problem $y' = \frac{x}{y}, y(0) = 1$ at $x = 0(0.4)2$ using fourth-order Runge Kutta method. If the exact solution is $y = \sqrt{x^2+1}$, find the absolute errors.

(10 marks)

(b) For a function *f*, the divided-differences are given as in **Table Q5(b)**.

Table Q5(b): Divided-differences for function *f*

$x_0 = 0.0$	$f(x_0) = ?$	$f_0^{[1]} = ?$	$f_0^{[2]} = 7.5$
$x_1 = 0.4$	$f(x_1) = ?$	$f_1^{[1]} = 12$	
$x_2 = 0.7$	$f(x_2) = 7$		

(i) Determine the missing entries in **Table Q5(b).**

(6 marks)

(ii) Determine the value of $f(0.25)$.

(2 marks)

(iii) Your friend, Luqman Hakim claims that the above data can be used to estimate $f(0.85)$. Do you agree with him? Justify your answer.

(2 marks)

Q6 **Figure Q6** shows a spring-mass system composed of three masses suspended vertically by a series of springs. To develop a mathematical model of the spring-mass system, Newton's second law can be employed in conjunction with force balances. For each mass, the Newton's second law can be expressed as:

$$m\frac{d^2x}{dt^2} = F_D - F_U$$

where m is the mass of an object, $\frac{d^2x}{dt^2}$ is the acceleration of an object, F_D is downward force and F_U is upward force. When the system eventually comes to rest (steady state), the displacements of the masses are expressed as

$$\begin{aligned} 3kx_1 - 2kx_2 \qquad\quad &= m_1 g \\ -2kx_1 + 3kx_2 - kx_3 &= m_2 g \\ - kx_2 + kx_3 &= m_3 g \end{aligned}$$

(a) Write the system of linear equations in complete matrix form.

(2 marks)

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(b) Solve the system of linear equations in **Q6(a)** using Doolittle method. Given $k = 10\text{kg/s}^2$, $g = 9.81\text{m/s}^2$, $m_1 = 20\,kg$, $m_2 = 250\,kg$ and $m_3 = 50\,kg$.

(8 marks)

(c) Compare the answer in **Q6(b)** with Crout method. Give conclusion to your finding.

(10 marks)

-END OF QUESTIONS-

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SEMESTER/SESSION	: SEM II / 2022/2023	PROGRAMME CODE	: BDD
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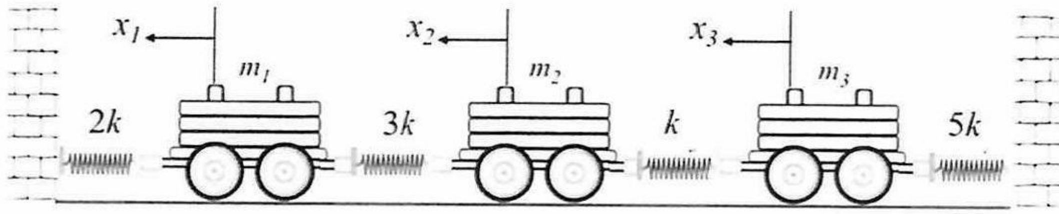
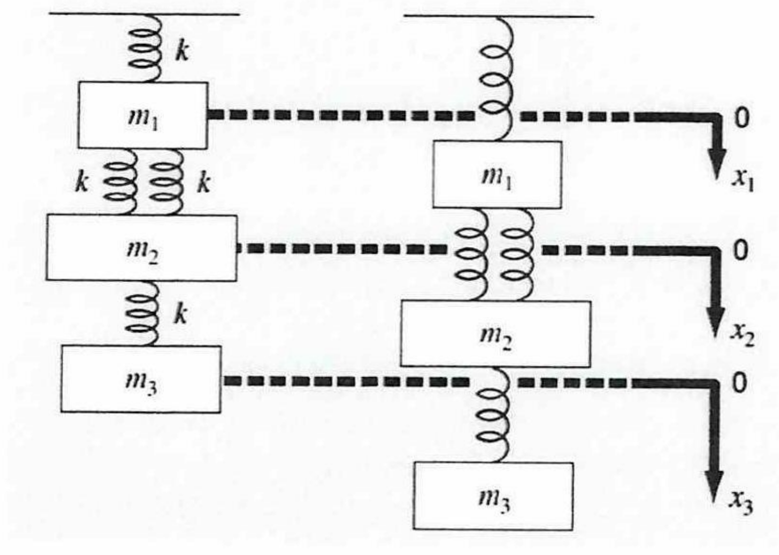


Figure Q1



The system before release The system after release

Figure Q6

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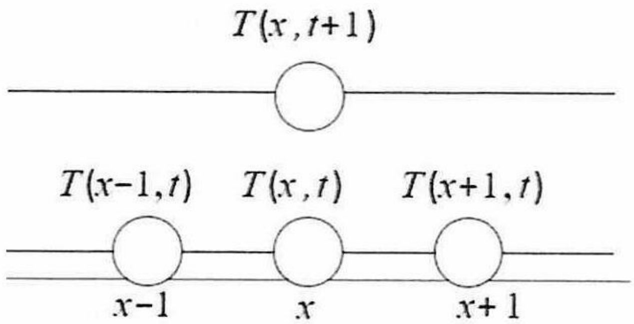
FINAL EXAMINATION

SEMESTER/SESSION	: SEM II / 2022/2023	PROGRAMME CODE	: BDD
COURSE NAME	: ENGINEERING MATHEMATICS IV	COURSE CODE	: BDA 34003

FORMULA

Explicit Method:



Euler 's Method:

$$y(x_{i+1}) = y(x_i) + y'(x_i)\,h$$

Power Method:

$$\{V\}^{k+1} = \frac{[A]\{V\}^k}{\lambda_{k+1}}$$

Inverse Power Method:

$$\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^k}{\lambda_{k+1}}$$

Characteristic Equation: det(A-λI)=0

3 Point Central Difference:

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h}$$

$$y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2}$$

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FORMULA

Bisection Method:

$$c = \frac{a+b}{2}$$

Secant Method:

$$x_{i+1} = \frac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$$

Newton Divided Difference:

$$y(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$

Fourth Order Runge Kutta Method:

$$f(x_i, y_i) = y'(x_i)$$

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$$

$$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$$

$$k_4 = hf(x_i + h, y_i + k_3)$$

Doolittle Method

$[A] = [L][U]$

$$\begin{pmatrix} 1 & 0 & \ldots\ldots 0 \\ l_{21} & 1 & \ldots\ldots 0 \\ \vdots & \vdots & \vdots \\ l_{n1} & l_{n2} & \ldots\ldots 1 \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & \ldots\ldots u_{1n} \\ 0 & u_{22} & \ldots\ldots u_{2n} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \ldots\ldots u_{mn} \end{pmatrix}$$

Crout Method

$[A] = [L][U]$

$$\begin{pmatrix} l_{11} & 0 & \ldots\ldots 0 \\ l_{21} & l_{22} & \ldots\ldots 0 \\ \vdots & \vdots & \vdots \\ l_{n1} & l_{n2} & \ldots\ldots l_{nn} \end{pmatrix} = \begin{pmatrix} 1 & u_{12} & \ldots\ldots u_{1n} \\ 0 & 1 & \ldots\ldots u_{2n} \\ \vdots & \vdots & \vdots \\ 0 & 0 & \ldots\ldots 1 \end{pmatrix}$$

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