CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BDU 11003
PROGRAMME CODE	:	BDC/BDM
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A** AND **ONE (1)** QUESTION **ONLY** IN **PART B.**
		2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED** BOOK.
		3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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PART A: ANSWER **ALL** QUESTIONS

Q1 (a) Solve the initial value problem.

$$y' - 2y = f(t),\ y(0) = 0 \qquad \text{where } f(t) = \begin{cases} 2, & 0 \le t < 2 \\ t, & t \ge 2 \end{cases}$$

(14 marks)

(b) Find the Laplace transform of each of the following functions.

(i) $f(t) = t \cosh \mathrm{t}$

(3 marks)

(ii) $f(t) = t^2 \sinh 4\mathrm{t}$

(3 marks)

Q2 (a) A periodic function $f(x)$ is defined by

$$f(x) = x, \quad -1 < x < 1$$

and $f(x) = f(x+2)$

(i) Sketch the graph of the function over $-3 < x < 3$.

(2 marks)

(ii) Find the Fourier coefficients corresponding to the function.

(9 marks)

(iii) Based on **Q2(a)(ii),** write the corresponding Fourier series.

(3 marks)

(b) Sketch the graph of each of the following function. Determine whether the function is even, odd or neither.

(i) $f(x) = \begin{cases} \cos x, & -\pi < x < 0 \\ 0, & 0 < x < \pi \end{cases}$

$f(x) = f(x + 2\pi)$

(3 marks)

(ii) $f(x) = \begin{cases} -1, & -2 < x < -1 \\ x, & -1 < x < 1 \\ 1, & 1 < x < 2 \end{cases}$

$f(x) = f(x + 4)$

(3 marks)

Q3 (a) Solve the following linear system using the Gauss elimination method by using the forward substitution technique.

$$x_1 + 2x_2 + x_3 + 4x_4 = 13$$
$$2x_1 + 0x_2 + 4x_3 + 34x_4 = 28$$
$$4x_1 + 2x_2 + 4x_3 + x_4 = 20$$
$$-3x_1 + x_2 + 3x_3 + 2x_4 = 6$$

(12 marks)

(b) Use the forward substitution method for solving the following linear system:

$$4x_1 = 8$$
$$2x_1 + x_2 = -1$$
$$x_1 - x_2 + 5x_3 = 0.5$$
$$0.1x_1 + 2x_2 - x_3 + 2x_4 = 12$$

(8 marks)

Q4 A rod of length 2m which is fully insulated along its sides, has an initial temperature distribution 100 sin $\left(\frac{1}{2}\pi x\right)°C$. At t = 0 the ends are dipped into iced and held at a temperature of $0°C$. The temperature distribution $u(x,t)$ satisfies the heat equation,

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2}{\partial x^2}$$

Determine the temperature distribution at a point P at a distance x from one end at any subsequent time t after $t = 0$.

(20 marks)

PART B: ANSWER **ONE** QUESTION **ONLY**

Q5 (a) Show that the solution is exact and find the general solution.

$$[e^{-y} + \cos(x-y) + 2x]dx - [xe^{-y} + \cos(x-y) + 1]dy = 0$$

(8 marks)

(b) Given $y'' - 2y' + 2y = e^{x}(1 + \sin x)$. Find the solution for the given differential equation by using the variation of parameters method.

(12 marks)

Q6 (a) Solve $x^2(1-y)\frac{dy}{dx} + y^2(1+x) = 0$ where $y(1) = 1$.

(8 marks)

(b) By using undetermined coefficients method, find the general solution for

$$y'' - 3y' + 2y = 10\sin 2x$$

(12 marks)

– END OF QUESTIONS –

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Formula's

Table 1: Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Table 2: Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Table 3: Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx$ $u_2 = -\int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Table 4: Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$f(t)H(t-a)$	$e^{-as}F(s+a)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s).G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$\dot{y}(t)$	$sY(s)-y(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$

Table 5: Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$