CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA **CLOSED BOOK**.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Sketch the graph of the following functions :

$$f(x)=\begin{cases} x^2-x+2 & ; x\le 1 \\ x^3+3 & ; 1<x\le 3. \\ 30 & ; x>3 \end{cases}$$

Hence, state the domain and range of $f(x)$.

(7 marks)

(b) Evaluate $\lim\limits_{x\to 4}\dfrac{\frac{8}{x}-2}{x-4}$.

(3 marks)

(c) Given that a function, $g(s)$ is written as:

$$g(s)=\begin{cases} \dfrac{s^2-9}{s-3} & ; s<3 \\ -(s^2)+r & ; 3\le s<9 \\ -31.3 & ; s=9 \\ 11-\dfrac{77}{9}s & ; s>9 \end{cases}.$$

(i) Compute the value of r if $\lim\limits_{s\to 3} g(s)$ exists.

(5 marks)

(ii) Determine whether the function is continuous at $s=9$.

(5 marks)

Q2 (a) Find $\dfrac{dy}{dx}$ of the given functions.

(i) $y=x^{-2}-\dfrac{\cos x}{5}+\dfrac{1}{2}\ln x$.

(4 marks)

(ii) $y=\ln x^2+\dfrac{e^x+2}{5}$.

(4 marks)

(b) Differentiate the following functions by using technique of differentiation.

(i) $y=\dfrac{x^3}{2\ln x+1}$.

(6 marks)

(ii) $y = \sqrt{3x^4 - 6\cos x}.$

(6 marks)

Q3 (a) Differentiate the following function using implicit differentiation method:

$$x^2 + y^3 = 8 - 3xy.$$

(4 marks)

(b) Using L'Hospital's Rule, evaluate $\lim_{x \to +\infty} f(x) = \dfrac{2 + 7x^2 - 4x}{6x - 5x^4}$.

(4 marks)

(c) Given a function as follows:

$$y = x^3 + \frac{3}{2}x^2 - 6x + 2.$$

(i) Determine the local extrema and fill out **Table Q3(c)(i)**.

(9 marks)

(ii) Hence, sketch the graph.

(3 marks)

Q4 (a) Evaluate the following integrals:

(i) $\displaystyle\int \left(2x^5 + 3\sqrt[3]{x} - \frac{4}{x^4} \right) dx.$

(4 marks)

(ii) $\displaystyle\int_0^4 \left(\frac{2\sqrt{x} + x^4}{x^2} \right) dx.$

(4 marks)

(b) Solve the integration using the given method.

(i) $\int x \sin 3x \, dx$ (part by part).

(6 marks)

(ii) $\int x^2 e^{2x} \, dx$ (tabular method).

(6 marks)

Q5 (a) **Figure Q5 (a)** shows the region R bounded by curve $y = 2x^2 + 10$ and line $y = 4x + 16$ that intersect at points A and B.

(i) Determine points A and B.

(4 marks)

(ii) Find the area enclosed by the curve and the line.

(5 marks)

(b) Use cylindrical shells to find the volume that results when the bounded region of $x + y = 2$, $x -$ axis, $y -$ axis is revolved about $x -$ axis.

(7 marks)

(c) Determine the length of the curve $y = \frac{(x^2 + 2)^{3/2}}{3}$ from $x = 3$ to $x = 4$.

(4 marks)

– END OF QUESTIONS –

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Table Q3(c)(i)

Critical and inflection points							
Test value							
f' behaviour							
f'' behaviour							

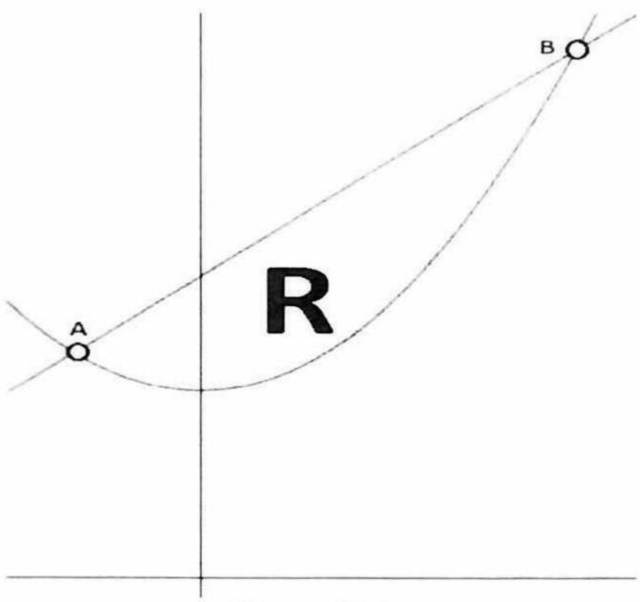

Figure Q5(a)

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LIST OF FORMULA

Table 1: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\ln\|u\|]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_a e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$
Chain Rule: $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$	Parametric Differentiation: $\frac{dy}{dx}=\left(\frac{dy}{dt}\Big/\frac{dx}{dt}\right)=\frac{dy}{dt}\cdot\frac{dt}{dx}$

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Table 2: Integration

$\int a\,dx = ax + C$	$\int \sin nx\,dx = -\frac{1}{n}(\cos nx) + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$	$\int \cos nx\,dx = \frac{1}{n}(\sin nx) + C$
$\int \frac{1}{nx+b}\,dx = \frac{1}{n}\ln\lvert nx+b\rvert + C$	$\int \tan x\,dx = \ln\lvert \sec x\rvert + C$
$\int \frac{1}{b-nx}\,dx = -\frac{1}{n}\ln\lvert b-nx\rvert + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int e^{nx}\,dx = \frac{1}{n}\left(e^{nx}\right) + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int e^{nx+b}\,dx = \frac{1}{n}e^{nx+b} + C$	$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$
Integration part by part: $\int u\,dv = uv - \int v\,du$	
Improper Integral: $\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$	
Identity: $1 + \tan^2 x = \sec^2 x$	

Area of Region

$$A = \int_a^b [f(x) - g(x)]\,dx \qquad \text{or} \qquad A = \int_c^d [w(y) - v(y)]\,dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\, f(x)\, dx \qquad \text{or} \qquad V = \int_c^d 2\pi y\, f(y)\, dy$$

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Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Partial Fraction

$$\frac{a}{x^2 - 1} = \frac{A}{x+1} + \frac{B}{x-1}$$