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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAC 11103
PROGRAMME CODE	:	DAA
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS.

2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**.

3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA **CLOSED BOOK**.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Solve the value of x and value of y in the following equation, given further that $x \in \mathbb{R}$ and $y \in \mathbb{R}$.

$$(x + iy)(2 + i) = 3 - i.$$

(6 marks)

(b) Given $z = -3 + 4i$ and $zw = -14 + 2i$. Find w in the form $a + bi$ and then express w in polar form.

(8 marks)

(c) Let z be a complex number such that $|z| = 4$ and $\arg z = \frac{5\pi}{6}$. Determine all roots of z^3.

(6 marks)

Q2 (a) Find the position vector given that vector $\mathbf{v}$ has an initial point at $(-3, 2)$ and terminal point at $(4, 5)$, then graph both vectors in the same plane.

(4 marks)

(b) Given $\mathbf{u} = \langle 3, -2 \rangle$ and $\mathbf{v} = \langle -1, 4 \rangle$. Compute a vector $\mathbf{w} = 3\mathbf{u} + 2\mathbf{v}$.

(3 marks)

(c) Given $\mathbf{v}_1 = 5i + 2j$ and $\mathbf{v}_2 = 3i + 7j$.

(i) Evaluate the dot product of $\mathbf{v}_1$ and $\mathbf{v}_2$.

(2 marks)

(ii) Determine the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$.

(4 marks)

(d) Find the magnitude and direction of the vector with initial point $\mathrm{P}(-8, 1)$ and terminal point $\mathrm{Q}(-2, -5)$.

(7 marks)

Q3 (a) Write down the first four terms of the binomial expansion of $(1 - 2x)^5$.

(4 marks)

(b) (i) Show that the equation $\tan 2x = 5\sin 2x$ can be written in the form $(1-5\cos 2x)\sin 2x = 0$.

(4 marks)

(ii) Hence, solve $\tan 2x = 5\sin 2x$ where $0° \le x \le 180°$. Giving the answer to 1 decimal places.

(5 marks)

(c) Solve the equation $4\cos^2 x + 7\sin x - 7 = 0$ where $0° \le x \le 180°$.

(7 marks)

Q4 (a) By using properties of real numbers, simplify the following expressions:

(i) $\frac{4}{7} \cdot \left(\frac{2}{3} \cdot \frac{7}{4} \right)$.

(2 marks)

(ii) $100\left[0.75 + (-2.38)\right]$.

(2 marks)

(b) Find the values of x that satisfies the equation:

$$\ln(2x+1) - \ln(x+3) = \ln 2x + \ln 3.$$

(4 marks)

(c) Simplify the following radical expression:

(i) $\sqrt{12}\sqrt{3}$.

(2 marks)

(ii) $\sqrt{\frac{5}{36}}$.

(2 marks)

(d) Expand $(x+4)(3x-2y+5)$.

(3 marks)

(e) Divide and express the quotient in simplest form:

$$\frac{2x^2+x-6}{x^2-1} \div \frac{x^2-4}{x^2+2x+1}.$$

(5 marks)

Q5 (a) Solve the following system of linear equations by using inversion method:

$$\begin{aligned} 2x - 17y + 11z &= 1 \\ -2x + 11y - 7z &= -1. \\ 3y - 3z &= 3 \end{aligned}$$

(11 marks)

(b) Solve the following system of linear equations by using Gauss-Jordan elimination method:

$$\begin{aligned} 2x + y - 2z &= 10 \\ 3x + 2y + 2z &= 1\ . \\ 5x + 4y + 3z &= 4 \end{aligned}$$

Perform the following operations in order:

$\frac{1}{2}R_1$,

$R_2 - 3R_1$,

$R_3 - 5R_1$,

$2R_2$,

$R_3 - \frac{3}{2}R_2$,

$-\frac{1}{7}R_3$,

$R_2 - 10R_3$,

$R_1 + R_3$,

$R_1 - \frac{1}{2}R_2$.

(9 marks)

- END OF QUESTIONS -

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Formula

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \quad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \quad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \quad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}, r > 1 \text{ or } S_n = \frac{a(1 - r^n)}{1 - r}, r < 1, \quad S_\infty = \frac{a}{1 - r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \dots\dots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

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$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \ |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = (c_{ij})^T \qquad A^{-1} = \frac{1}{|A|}Adj(A)$$

Vector

$$\cos\theta = \frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} \quad \text{or} \quad \boldsymbol{a}\bullet\boldsymbol{b} = x_1x_2 + y_1y_2 + z_1z_2, \ \boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$|a| = \sqrt{x^2+y^2+z^2}$, $x = x_0 + a_1 t$, $y = y_0 + a_2 t$, $z = z_0 + a_3 t$ and $\dfrac{x - x_o}{a_1} = \dfrac{y - y_o}{a_2} = \dfrac{z - z_o}{a_3}$

$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$

Complex number

$r = \sqrt{x^2+y^2} \qquad \tan\theta = \dfrac{y}{x}$

If $z = re^{i\theta}$, then $z^n = r^n e^{in\theta}$

If $z = re^{i\theta}$, then $z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta+2k\pi}{n}\right)i}$.

If $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

If $z = r(\cos\theta + i\sin\theta)$ then $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{(\theta+2k\pi)}{n} + i\sin\dfrac{(\theta+2k\pi)}{n}\right)$